

Damage Detection and Condition Monitoring of Pre-stressed Concrete Bridges by using Vibration-based Health Monitoring Techniques

K.S.D.M. Fernando, D.M.C.D. Dissanayake, M.A.K.M. Dharmasiri and A.J. Dammika

Abstract: Damage detection plays a vital role in bridges as the structural performance of the bridges decreases progressively throughout their service life due to many deterioration processes. Although damage detection is performed mainly through periodic visual inspection, due to its immense drawbacks, researchers have focused increasing attention to the potential of using vibration-based structural health monitoring (V-SHM) techniques to detect damages and monitoring conditions of the structures. The fundamental theory of the V-SHM is that structural damages will change mass, stiffness and damping properties of structures leading to detectable changes in their dynamic characteristics such as natural frequencies, mode shapes and modal damping ratios. The applicability of V-SHM techniques to detect the existence of damage in pre-stressed concrete bridges by experimentally identifying their dynamic characteristics is reported in this study. Eigensystem Realization Algorithm (ERA) was used for experimental modal analysis of the bridges by using the free vibration records extracted from field vibration measurements. Numerical modal analysis for the bridges was performed using finite element models of the bridges to obtain the dynamic characteristics of the intact structures. The identified dynamic characteristics from both experimental and numerical approaches were used in obtaining an indication of the possible level of existence of damages in studied pre-stressed concrete bridges. Three global damage identification (DI) criterions are defined and evaluated to further corroborate that these bridges have not undergone appreciable damages, as the evaluated DI criterions are within acceptable tolerances for no damage state, accommodating noisy data.

Keywords: PC bridges, Damage detection, Modal damping ratio, Experimental modal analysis

1. Introduction

Pre-stressed Concrete (PC) bridges are an essential component in Sri Lankan transportation network. These bridges are susceptible to damages due to the increase in traffic volume of the road network and adverse dynamic loading conditions in challenging environmental conditions. The structural performance of PC bridges decreases progressively throughout their service life due to deterioration processes such as fatigue, creep, reinforcement/pre-stress tendon corrosion and adverse loading. Hence, damage identification is required to maintain the infrastructure in a serviceable state and diagnose damage at an early stage so as to avoid disastrous failures.

Bridge damage detection is performed mainly through periodic visual inspection, which is often subjective. Invisible damages cannot be easily identified by visual inspection alone. To address these drawbacks, researchers have focused the attention increasingly on Vibration-based Structural Health Monitoring (V-SHM)

technique due to it being a non-destructive method of evaluation non-reliant on subjective human input such as visual inspection.

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Structural Health Monitoring (SHM) is defined in [1] as “at the simplest level, recurrent visual observation and assessment of structural condition (cracking, spalling and deformations) could be viewed as SHM, yet the aim of present-day research is to develop effective and reliable means of acquiring, managing, integrating and interpreting structural performance data for maximum useful information at a minimum cost while either removing or supplementing the qualitative, subjective and unreliable human element”, which sums up the definition of SHM quite well.

The fundamental theory behind vibration-based damage identification technique is that damage induces changes in physical properties of the structures resulting in detectable changes in modal properties such as natural frequencies [2], mode shapes [3] and modal damping ratios [3]. This implies that modal parameters vary according to the extent of damage and identifying the differences/trends in the variation of modal parameters with damage may give reliable insights to measure the level of damage of structures.

The research data presented here will be beneficial to understand the feasibility of adopting the V-SHM techniques to detect structural damages by identifying the dynamic characteristics including natural frequency, mode shapes and modal damping ratios of the studied PC bridges.

Hence, this research was carried out to identify the damage level of two existing single span PC bridges by using the V-SHM techniques. Field vibration measurements of the two single spanned PC bridges were taken under normal service conditions by using a wired accelerometer setup. Experimental modal identification was done by Eigensystem Realization Algorithm (ERA) and numerical modal parameter identification was done by the finite element models created using commercially available finite element software package ‘SAP2000’. For one of the PC bridges, the measurements recorded in two consecutive years (2017 and 2018) were analyzed and results were reported while the results presented for the other bridge was based on measurements recorded in 2017 only. The identified dynamic characteristics from both experimental and numerical approaches were used to establish the level of existence of damages in the studied PC bridges. To this end,

three damage identification (DI) criteria were defined and evaluated. From the damage assessment process, which involves four levels, i.e., damage detection, damage localization, damage quantification and damage consequence (assessment of remaining service life of the structure) [4], the study presented herein is focused on damage detection of PC bridges only.

2. Literature Review

V-SHM techniques can be categorized as non-physical and physically based methods. Non-physically based approach uses a system model to study a physical structure and predict the responses. The physically based detection method identifies the damage by comparing the modal parameters between the healthy and damaged structural model. A well-known classification for the damage identification methods, presented by [4] defines four levels of structural health monitoring.

Level 1: Damage detection – determination that damage is present in the structure.

Level 2: Damage localization – determination of the geometric location of the damage.

Level 3: Quantification of the severity of the damage.

Level 4: Prediction of the remaining service life of the structure.

Based on the features used for damage identification, the damage detection methods can be classified as natural frequency based methods, mode shape and curvature based methods, and damping change based methods. The simplicity of natural frequency based methods makes it attractive as a means of damage detection. Moreover, natural frequencies can be conveniently measured and used as a parameter for modal validation [5]. However, mode shapes are more sensitive to local damages and this enables them to be used directly in multiple damage prevalence scenarios [5]. Furthermore, they are less sensitive to environmental effects when compared with natural frequency [5]. Modal damping ratio is also a very sensitive indicator. By means of observing damping changes one has the ability to detect the nonlinear, dissipative effects that cracks produce [6].

Eigensystem Realization Algorithm (ERA) [7] is a high precision system realization method which uses the time domain response of multi-input and multi-output (MIMO) data to

identify modal parameters. That is, modal parameters extracted from ERA utilize multiple synchronous outputs (time domain data) simultaneously to arrive at a high precision estimate for the modal parameters. In this study, as several sensors were used for synchronous measurement of the acceleration of the bridge, ERA was one of the most appropriate methods to extract modal parameters. This method exhibits important features, such as the robustness of the algorithm requiring only simple numerical operations, numerical stability, the ability to use data from more than one test simultaneously to efficiently identify closely spaced eigenvalues, and moderate computation requirements, which encouraged the use of ERA in this study.

3. Methodology

The methodology consists of both experimental and numerical analysis to identify the modal parameters including natural frequencies, mode shapes and damping ratios. The experimental part of the study involved the measurement of structural response induced under normal service condition of two bridges and identification of dynamic characteristics (natural frequencies, mode shapes and damping ratios) by analyzing the acceleration data obtained from the field vibration measurements. For one of the bridges, results were analyzed and reported for two consecutive years. The experimental modal parameter identification was done by analyzing the extracted free vibration records using the Eigensystem Realization Algorithm [7].

In the numerical study, the eigenvalue analysis of the finite element models of the two PC bridges was conducted using a commercially available software package (SAP2000) to obtain the dynamic characteristics of the structures.

3.1 Description of the Bridges

Two single spanned PC bridges, bridge No 43/10 and bridge No 13/3, were selected for this study as shown in Figure 1 and Figure 2, respectively. Bridge No 43/10 is located in the Kandy-Mahiyaganaya-Padiyathalawa Road (A26), Hunnasgiriya, Sri Lanka which was constructed in 2012, while bridge No 13/3 is situated in the Hindagala-Naranwita-Gampola Road (B154), Doluwa, Sri Lanka which was constructed in 2000. The span details of the studied PC bridges are shown in Table 1. The

bridge 13/3 consists of 14 pre-stressed concrete girders while bridge 43/10 consists of 19 pre-stressed concrete girders.

Table 1 - Bridge Span Details

| Bridge | Length(m) | Width(m) |
|--------------|-----------|----------|
| Bridge 43/10 | 13.0 | 10.35 |
| Bridge 13/3 | 16.2 | 8.40 |

3.2 Accelerometer Positioning

The accelerometer sensors ("Guralp Fortis" 0~200 Hz) were placed at optimal locations (maximum modal amplitude locations of the first two modes) to capture the first two mode shapes of the bridges effectively by experimental modal analysis. The sensor setup for bridge 13/3 and bridge 43/10 are shown in Figure 3 and Figure 4, respectively.

These accelerometers were connected to a data logger ("Tokyo Sokki Kenkyujo DC-204Ra") to record and save the resultant acceleration-time history data from vibration caused by vehicles passing through the bridge, for further analysis.

3.3 Field Vibration Measurement

The vertical vibration measurements of the two single spanned PC bridges were performed during regular service conditions by using a wired accelerometer setup. The sampling rate of the sensor setup was set to 200 Hz. All accelerometers were switched on at the same time when the vehicles were about to enter the bridge. The movement of the vehicles on the bridge induces vibration in the bridge due to the dynamic nature of the load applied. These vibrations usually do not immediately cease after the vehicles exit the bridge but continue to vibrate until the energy is dissipated. This brief period after which the vehicles have exited the bridge but the bridge is continuing to vibrate albeit with diminishing amplitude can be regarded as free vibration. These free vibrations are of interest in this study. A stopwatch was used to determine the time measurements required to extract free vibration data from the vibration records. At the same time, a video was recorded to verify the data later. The accelerometers were turned off a few seconds after the vehicles left the bridge.

For bridge 43/10, the field vibration measurements were conducted in 2017 while for bridge 13/3, they were carried out in 2017 and 2018.



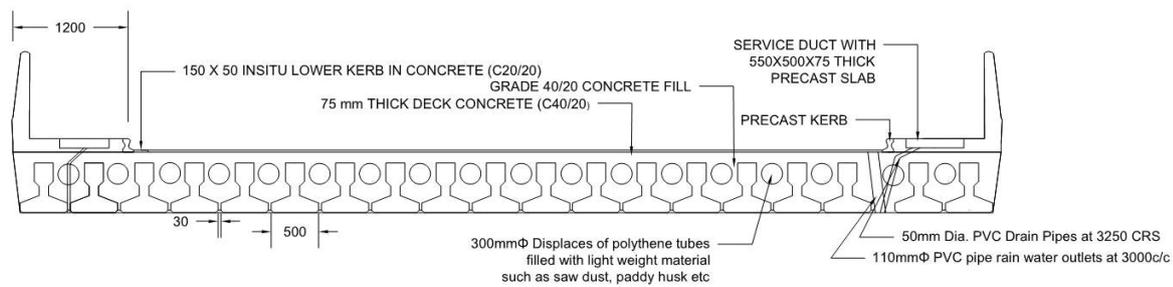


Figure 1 - Section of the Bridge 43/10 (Units are in Millimeters)

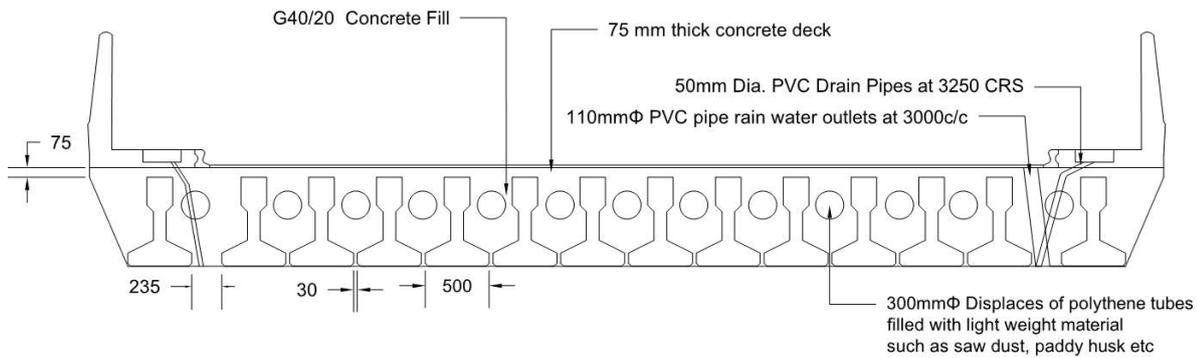


Figure 2 - Section of the Bridge 13/3 (Units are in Millimeters)

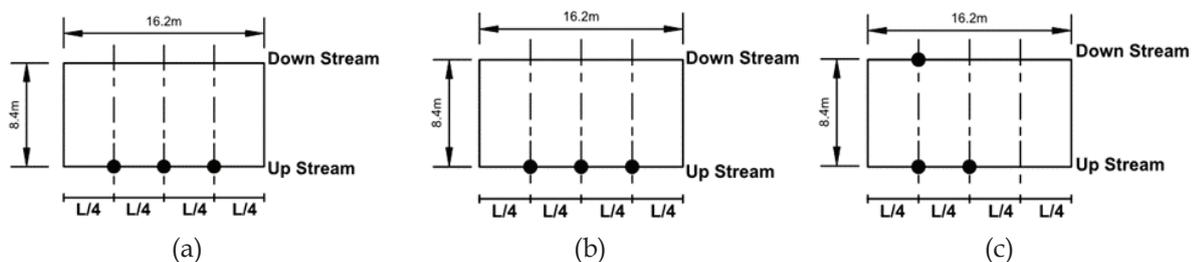


Figure 3 - Sensor Setup for the Bridge 13/3 (a) 2017 Analysis, (b) and (c) 2018 Analysis

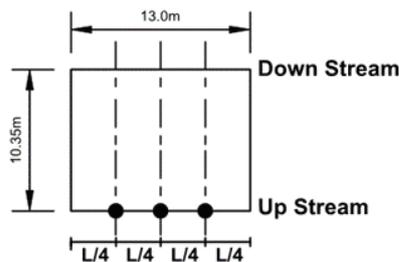


Figure 4 - Sensor Setup for the Bridge 43/10

3.4 Experimental Modal Identification

For the experimental modal identification, Eigensystem Realization Algorithm (ERA) was applied on the vibration records obtained from ambient vibration monitoring of the bridges. From these vibration records, data belonging to the free vibration period was extracted so that they could be used in the experimental modal identification with ERA. In Figure 5, a sample of extracted free vibration time histories of bridge 13/3 (obtained in 2017) is illustrated.

3.5 Modal Identification Steps in ERA

Field measured vibration records were used as inputs to ERA while natural frequencies and modal damping ratios are the main output. The steps involved in the modal identification process in ERA are illustrated in Figure 6.

Results were subjected to step by step screening process in order to increase the accuracy of the results. Since ERA results usually fluctuate over different system orders, persistent results across system orders were considered as stable robust results. In ERA, physical vibration modes were identified by applying a screening process based on several criterions. The Modal Amplitude Coherence (MAC) [7] and the Extended Modal Amplitude Coherence (EMAC) [8] were used in this study for screening the results which were identified by ERA. Unstable spurious modes were removed by these screening criterions.

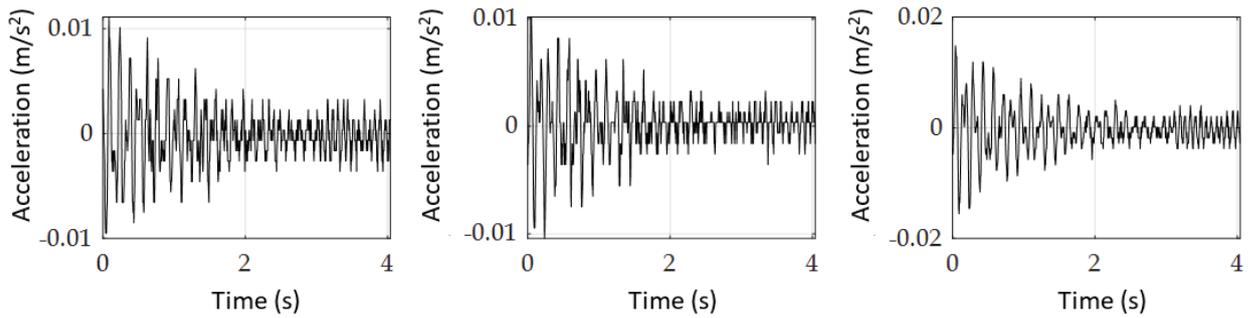


Figure 5 –A Sample of Extracted Free Vibration Record (Bridge 13/3, 2017 – at the Three Accelerometer Sensor Locations)

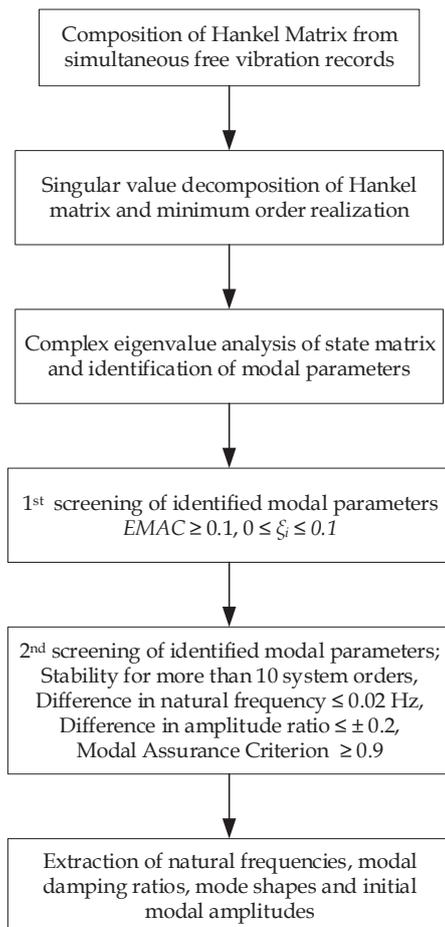


Figure 6 – Steps involved in ERA with the Applied Screening Criteria

In the screening process, a stabilization diagram is used to show the stability of identified modes against system order. A sample of stabilization diagrams before and after screenings is shown in Figure 7 and Figure 8, respectively. The conditions used for screening are as follows. These screening criterions are typical values

used in ERA analysis, accommodating the noisy conditions of data.

MAC threshold = 0.9
 EMAC threshold = 0.1
 Damping Ratio Upper threshold = 0.1
 Damping Ratio Lower threshold = 0.0

3.6 Experimental Modal Parameter Identification

Natural frequencies were determined for each of the free vibration data samples using the outputs obtained from the ERA analysis. Fast Fourier Transformation (FFT) was applied on the free acceleration-time history data obtained from the field measurements to verify the natural frequency results obtained from ERA.

Modal damping ratios were determined for the two PC bridges using the results obtained through ERA and refined by clustering against system order as well as natural frequencies. That is, the damping ratios obtained from all data sets were compared with the system order, thereby removing outliers in estimation and arriving at an average value considering the stable range of system orders.

The variation of the damping ratio with system order is plotted in Figure 9 for a particular measurement data set. The variation of damping ratio with natural frequency is illustrated in Figure 10 and Figure 11 for bridge 43/10 and bridge 13/3, respectively.

Mode shapes for the two bridges were obtained for a particular mode for a particular damping ratio.



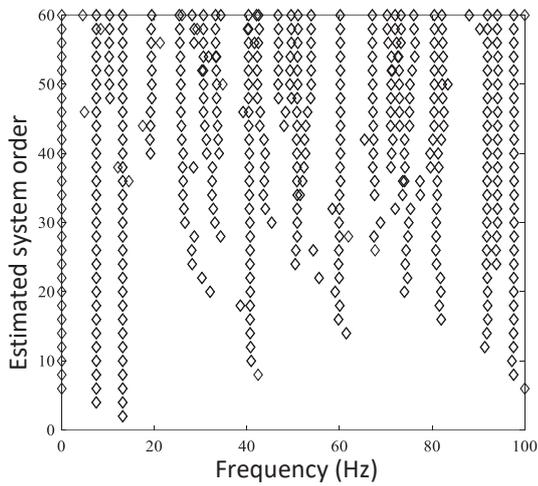


Figure 7 - Original Stabilization Diagram before Applying Screening Criteria (Bridge 13/3, 2017)

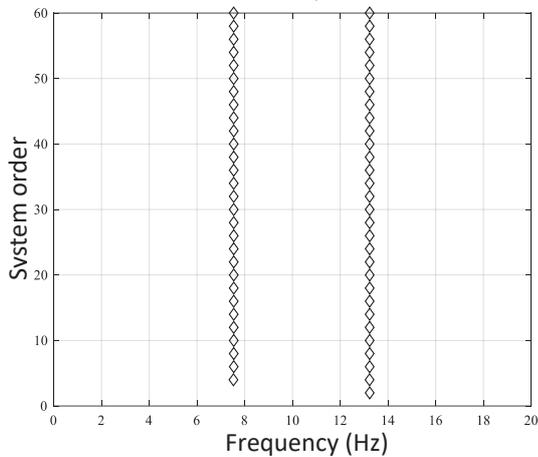


Figure 8 - Refined Stabilization Diagram after Applying Screening Criteria (Bridge 13/3, 2017)

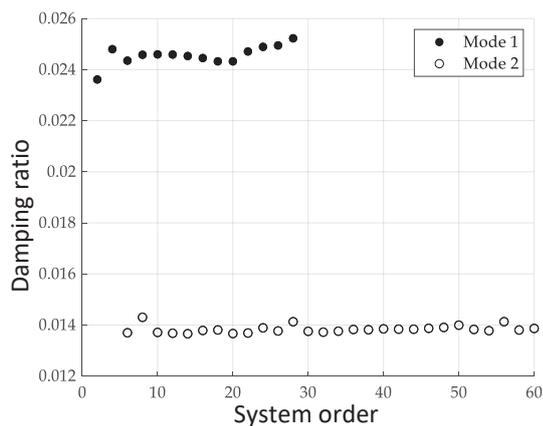


Figure 9 - The Variation of Damping Ratio with System Order - Bridge 43/10

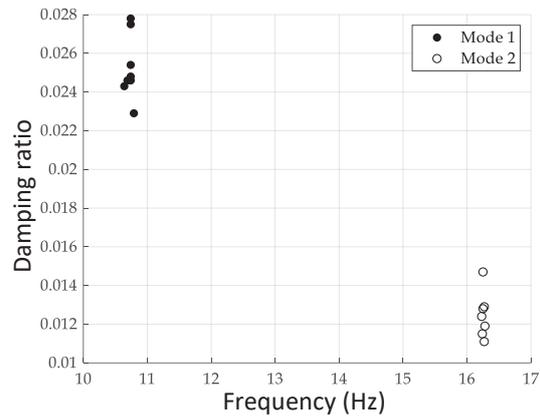


Figure 10 - The Variation of Damping Ratio with Natural Frequency - Bridge 43/10

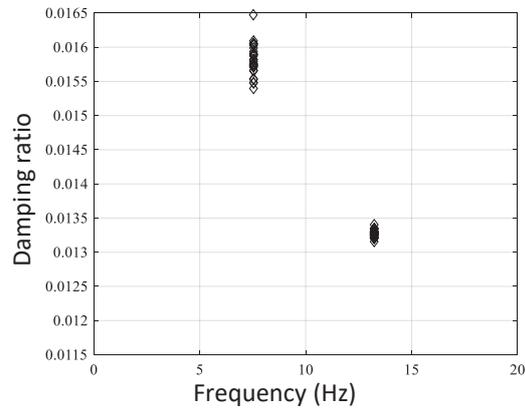


Figure 11 - The Variation of Damping Ratio with Natural Frequency - Bridge 13/3

3.7 Numerical Modal Parameter Identification

The numerical modal parameter identification of the two PC bridges considered was performed by using finite element models (FEM), modelled using commercially available software SAP2000. The required modal characteristics were obtained from the eigenvalue analysis results. Only the superstructure of the studied bridges was modelled including pre-stressed girders and deck. Further details related to the FE modeling are shown in Table 2.

As for the boundary conditions, fixed and pin support conditions were used to represent the support conditions of the bridges on either side of the span.

3.8 Damage Identification

Natural frequency, mode shapes and modal damping ratios can be used as major modal parameters of the PC bridges for the process of damage identification. Mode shapes obtained from experimental analysis and numerical modal analysis were compared to identify the damages. Two sets of mode shapes obtained

from the ERA analysis and the FEM were compared using the Modal Assurance Criterion (MAC) [9].

$$MAC(i, j) = \frac{|\phi_{ERA}^i{}^T \phi_{FEM}^j|^2}{|\phi_{ERA}^i{}^T \phi_{ERA}^i| |\phi_{FEM}^j{}^T \phi_{FEM}^j|} \quad \dots(1)$$

where ϕ is the mode shape vector of the respective analysis mode.

The equation (1) was used to obtain the MAC value of a particular mode shape i with respect to a mode shape j . If MAC is equal to 1, it is the perfectly correlated condition and 0 is for no correlation condition. From these MAC values, damage level can be identified.

Modal damping ratio can also be used as an effective and sensitive damage identification modal parameter [10]. Since damping ratios cannot be identified by the numerical model, results from similar bridges (approximately similar in geometrical and structural) or the same bridge but at different time frames can be used for damage detection.

3.9 Damage Identification Criteria

Three damage identification (DI) criteria based on (1) natural frequency, (2) mode shapes and (3) modal damping ratios are defined in this study to predict the prevalence of damages in the bridges. These DI criteria are global metrics, i.e. no damage localization is possible using these indices.

The DI criterion based on natural frequencies is defined as,

$$DI_f = \left| 1 - \frac{f_d}{f_u} \right| \quad \dots(2)$$

Where f_d and f_u are the natural frequencies identified from the experimental modal identification and numerical modal identification, respectively.

The DI criterion based on MAC value (mode shape representation) of a particular mode shape is defined as,

$$DI_s = 1 - MAC_d \quad \dots(3)$$

where MAC_d is the MAC value computed using equation (1) for a particular mode shape.

The DI criterion based on modal damping ratios is defined as,

$$DI_d = \left| 1 - \frac{\zeta_d}{\zeta_u} \right| \quad \dots(4)$$

where ζ_d and ζ_u are the modal damping ratios computed from the experimental modal identification at two different time frames.

4. Results and Discussion

Results of both bridges are summarized in Table 3 and Table 4. All modes of vibrations which were stably identified using ERA which have the least co-efficient of variance were considered for the mode shape identification. By using the finite element models, numerically identified values for the natural frequency and its corresponding mode shapes were found. FFT of free vibration records were also obtained for each and every vibration measurement. Results obtained from FFT and ERA for the two modes are shown in Table 3. Frequency identifications from all three methods, namely FFT, ERA and FEM analysis, for one set of field measurement are plotted in the same figure

Table 2 – Numerical Analysis FEM Details

| Bridge | Structural Element | Finite Element Used | Material | Material Properties | | | Remarks |
|------------------------------|--------------------|---------------------|----------|---|-----------------------------|-----------------|--|
| | | | | Weight per unit volume (kN/m ³) | Modulus of Elasticity (GPa) | Poisson's Ratio | |
| Bridge 43/10 and Bridge 13/3 | Girder | Frame | C45/55 | 24.99 | 36 | 0.2 | Support conditions: Fixed-Pin Added mass loads: Pavement asphalt layer, handrails, pedestrian walkway and other components. |
| | Deck Slab | Four-node Shell | C40/50 | 24.99 | 35 | 0.2 | |
| | Tie Rods | Frame | Steel | 76.90 | 200 | 0.3 | |



(Figure 12) to verify the accuracy of the identified modes. Due to the malfunctioning of an accelerometer sensor (third sensor from the left, Figure 4) in the investigation of bridge 43/10, the data from that particular sensor was not used in the analysis.

Mode shapes obtained from both experimental and numerical analysis are shown in Table 4. For bridge 43/10 and bridge 13/3, natural frequencies were identified stably using ERA and FFT and corresponded with the numerical model, which implies successful model validation.

For both bridges, the symmetric vertical bending mode and asymmetric vertical bending modes were identified with higher stability. The Modal Assurance Criterion (MAC) calculated between mode shapes identified from both experimental and numerical approaches for both bridges bear witness that the numerical models for both bridges are well correlated with the physical structures and the bridges are devoid of appreciable damage.

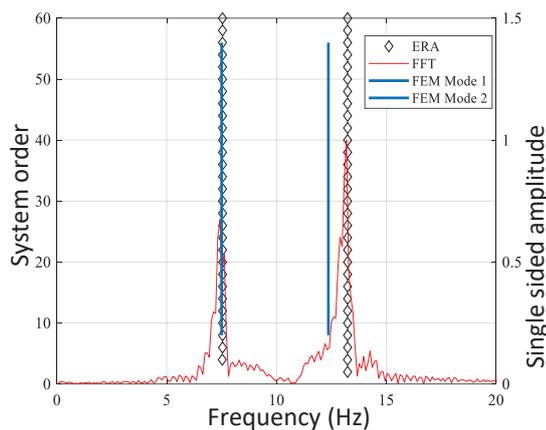


Figure 12 - Comparison of Frequency Identifications from ERA, FFT and FEM (Bridge 13/3)

The modal damping ratios were also stably identified for both bridges through this experimental approach and they can be used for damage detection of the same bridges in the future.

Furthermore, the DI criteria defined in section 3.9 were evaluated for the two bridges and tabulated in Table 5. The experimental modal analysis results used to compute the DI criteria were obtained from the ERA analysis while the numerical modal analysis results used to compute the DI criteria were obtained from the eigenvalue analysis using commercially available software package “SAP 2000”. The latter results were used to represent the parameters of the intact structure except in the case of the computation of DI_d where the reference base values were taken from the identification conducted in 2017 for bridge 13/3.

Considering the bridge 43/3, the DI criteria DI_f and DI_s are within the tolerance for no damage state, accommodating noisy data. As such, it can be inferred that the bridge is devoid of appreciable damages. Since no multiple investigations were conducted for the bridge 43/3, the DI criterion based on damping ratios could not be computed.

For the bridge 13/3, investigations were conducted for consecutive years and, as such, all the three DI criteria defined in Section 3.9 were computed. All the DI criteria are within acceptable tolerances for no damage state, accommodating noisy data, implying no appreciable damage.

Table 3 - Dynamic Characteristics of Bridge 43/10 and Bridge 13/3

| Bridge 43/10 | | | | | |
|--------------|-----------------------|-----------------------|--------|---------------|---------------|
| Mode | Natural Frequency(Hz) | | | Damping Ratio | |
| | FFT | ERA | FEM | | |
| Mode 01 | 10.72 | 10.73 | 10.34 | 0.0252 | |
| Mode 02 | 16.17 | 16.26 | 16.38 | 0.0111 | |
| Bridge 13/3 | | | | | |
| Mode | Year | Natural Frequency(Hz) | | | Damping Ratio |
| | | FFT | ERA | FEM | |
| Mode 01 | 2017 | 7.457 | 7.522 | 7.484 | 0.0157 |
| | 2018 | 7.460 | 7.457 | | 0.0152 |
| Mode 02 | 2017 | 13.117 | 13.202 | 13.231 | 0.0132 |
| | 2018 | 13.232 | 13.235 | | 0.0135 |

Table 4 – Mode Shapes Obtained from ERA Analysis and FEM Analysis

| Modes | Bridge 43/10 | | | | |
|---------|--------------|-----------|-----------|--------|--------|
| | Experimental | Numerical | MAC | | |
| Mode 01 | | | 0.9886 | | |
| Mode 02 | | | 0.9916 | | |
| Modes | Bridge 13/3 | | | | |
| | Experimental | | Numerical | MAC | |
| | 2017 | 2018 | | 2017 | 2018 |
| Mode 01 | | | | 0.9955 | 0.9849 |
| Mode 02 | | | | 0.9961 | 0.9963 |

Table 5 – Damage Identification (DI) Criteria for the Two Bridges

| Bridge 43/10 | | | | | |
|--------------|----------------------------|-----------------------------|--------------------------------|--------------------------------|---|
| Mode | DI criterion | | | Remarks | |
| | Frequency based (DI_f) | Mode shape based (DI_s) | Damping ratio based (DI_d) | | |
| Mode 01 | 0.0377 | 0.0114 | - | - | |
| Mode 02 | 0.0073 | 0.0084 | - | - | |
| Bridge 13/3 | | | | | |
| Mode | Year | DI criterion | | | Remarks |
| | | Frequency based (DI_f) | Mode shape based (DI_s) | Damping ratio based (DI_d) | |
| Mode 01 | 2017 | 0.0051 | 0.0045 | - | Base year for calculation of DI_d is considered as 2017 |
| | 2018 | 0.0036 | 0.0151 | 0.0318 | |
| Mode 02 | 2017 | 0.0022 | 0.0039 | - | |
| | 2018 | 0.0003 | 0.0037 | 0.0227 | |

5. Concluding Remarks and Recommendations

An approach to detect damages by using vibration-based health monitoring techniques is presented. In this study, the general procedure involved in the V-SHM, which involves establishing and validating a numerical model

under no damage conditions and identifying the modal characteristics of the physical structure (bridge) using experimental modal identification (in this case using ERA), are done. Then the comparison of numerically obtained modal parameters with those obtained from experimental investigation to identify the presence of damage is presented using two case studies. Further more, three global DI criterions



are defined and evaluated to assess the presence of damage in the two bridges. The variation of the DI criteria over the years would be worthwhile to investigate, to identify the effect of noise in the measurements and subsequently identify the presence of damage.

The experimental modal parameter identification results and the modal analysis results from the numerical model are within acceptable tolerances. Therefore, it is prudent to state that the two vibration modes were successfully identified for both pre-stressed concrete bridges, affirming that this technique can be applied to identify modal parameters experimentally.

The evaluated DI criteria further affirm that the bridges have not undergone appreciable damage. The defined global DI criteria based on natural frequencies, mode shapes and modal damping ratios can be used to identify damages in future investigations.

For damping based damage identification methods, stable damping ratio identifications are a necessity which require high quality structural vibration response measurement using highly sensitive sensors together with stable damping identification techniques. This study proves the capability of stable damping identification; hence it can be recommended to be used for damage detection. Damage location identification is possible with mode shape-based methods and since an accurate depiction of the mode shape requires the establishment of modal amplitudes at as many locations as possible, this necessitates the use of properly located higher number of sensors for field vibration measurements.

Identified results in this study can be applied for damage detection of the same bridges in future, as well, to get an understanding on damping characteristics of the bridges with similar configurations.

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References

1. Brownjohn, J. M. W., 2007. "Structural Health Monitoring of Civil Infrastructure". *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 365, 589-622.
2. Wang, L. and Chan, T. H., 2009. "Review of Vibration-based Damage Detection and Condition Assessment of Bridge Structures using Structural Health Monitoring". In: *QUT Conference Proceedings*.
3. Moaveni, B., He, X., Conte, J.P. and Udd, E., 2006. "Effect of Damage on Modal Parameters using Full-Scale Test Data", in: *Proceedings of the International Conference on Modal Analysis (IMAC-XXIV)*, St. Louis, USA.
4. Rytter, A. "Vibrational Based Inspection of Civil Engineering Structures". Aalborg: Dept. of Building Technology and Structural Engineering, Aalborg University. *Fracture and Dynamics*, No. 44, Vol. R9314, 1993.
5. Fan, W. and Qiao, P., "Vibration-based Damage Identification Methods: A Review and Comparative Study", *Structural Health Monitoring*, 10(1), 2011, pp. 83-110.
6. Sohn, H., Farrar, C., Hunter, N. and Worden, K., "A Review of Structural Health Monitoring Literature: 1996-2001", Los Alamos National Laboratory Report (NM 87545, USA), Vol. LA-13976-MS, 2003.
7. Juang, J., and Pappa, R., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Modal Reduction", *Journal of Guidance*, 8(5), 1985, pp. 620-627.
8. Pappa, R., Elliott, K. and Schenk, A., 1992. "A Consistent-Mode Indicator for the Eigensystem Realization Algorithm", in: *the Dynamics Specialists Conference*, American Institute of Aeronautics and Astronautics, Dallas, TX, U.S.A.
9. Allemang, R. J. and Brown, D. L., "A Correlation Coefficient for Modal Vector Analysis", *Proceedings, International Modal Analysis Conference*, 1982, pp. 110-116.
10. Dammika, A., Kawarai, K., Yamaguchi, H., Matsumoto, Y. and Yoshioka, T., "Experimental-Analytical Framework for Damping Change-Based Structural Health Monitoring of Bridges" Research Report of Department of Civil and Environmental Engineering, Saitama University, Vol. 41, 2015.