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A New Approach to the Study of Voltage Distribution along a Suspension Insulator String

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Abstract: The cap and pin insulator string, though a simple element in a power system, has the responsibility of carrying the live conductor while insulating it properly from the tower arm. In properly designing the insulator string, it will be necessary to assess the voltage distribution along it to ensure that no unit will get unduly stressed beyond its capacity. There are standard methods available to determine the voltage distribution along an insulator string. However, all these methods have limitations. This paper describes a novel method for accurately computing the voltage distribution along a string insulator which can be used for either short or long strings with no restrictions whatsoever. The computation allows to accommodate the capacitance between the metal work of the string and the tower and the capacitance between the metal cap of the string and the guard ring. The paper also describes two new simple equations that can be used to determine the voltage distribution along an insulator string irrespective of whether it is short or long.

Keywords: Suspension insulator string, Voltage distribution, New method of calculation

1. Introduction

A high voltage line conductor is usually supported at the transmission tower arm by a string of suspension insulators. The string holds the conductor mechanically and also insulates it from the tower arm. It is formed by arranging in series, several standard cap and pin suspension insulator units or discs. Usually, the discs are made of porcelain or toughened glass with a metal cap on top and a pin at the bottom which is suitably made to engage with the cap of the next insulator. The number of units required in a string will depend on the system voltage level. Even though the insulator units are identical, the voltage distribution along the string will not be uniform. This is due to the capacitance effect between the metal work of the insulator units and the steel tower or arm and, to a lesser extent, due to the capacitance effect between the metal cap and the guard ring provided at the connection of the line conductor and the lowest insulator unit.

In the design of an insulator string, it is necessary to know how the voltage will be distributed among the several insulator units, when the line conductor has been charged to a particular voltage. This gives information to the designer to ensure that the units will not get electrically stressed beyond their limits. It is commonly known that the unit nearest the line conductor will be more stressed than any other unit forming the string.

There are standard methods to determine the voltage distribution along a string. However, these methods are cumber some and laborious for lines at 220kV, 400kV etc., for which the string will have to contain 12 or more units.

This paper describes several new methods that can be used to compute the voltage distribution along insulator strings for any voltage and for any number of insulator units in the string.

2. Standard Methods available for Voltage Distribution Calculations

In these calculations, it is assumed that the cap and pin of the insulator unit with porcelain or glass as its insulating material has a capacitance. It is also assumed that the insulator unit metal work and the tower metal work forms an air capacitance although it is of much lower value. In the same manner, it is assumed that the metal work of the insulator unit and the guard ring also forms an air capacitor.

Case 01

In an ideal situation, i.e., when the insulator is remote from other metal work and when only the capacitances of the insulator units or self-capacitances come into play, the voltage will be distributed equally among the units as shown in Figure 1 for a string insulator with 5 units.

The voltage across each unit will be V/5 where Vis the voltage of the line conductor with respect to earth.

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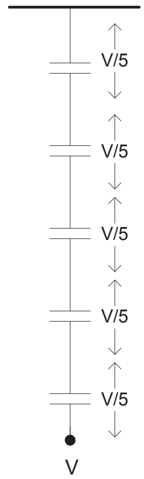


Figure 1 - Configuration for a five unit insulator string remote from the tower

Case 02

In this case, the effect of the tower is considered since there are self-capacitances of units and capacitance between the units and the tower and tower arm. This configuration is shown in Figure 2. In this case, the distribution of voltage will not be linear and the non-linearity will depend on the number of insulator units and the ratio of the capacitance between the unit and the tower to the self-capacitance. It is to be noted that the value of k is much less than 1, and that it ranges from 7 % to 20 %[1]. In the usual calculations, it is common to consider k=0.1, i.e., 10 %.

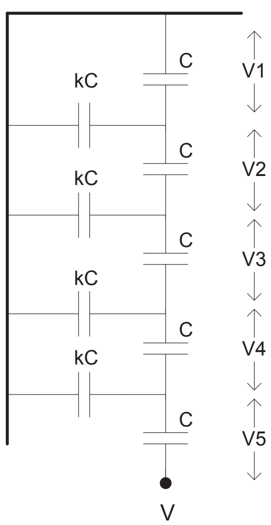


Figure 2 - Configuration for a five unit insulator string close to the tower

(a) Method of Calculation – Use of Kirchhoff's Laws

In this case where the units are in the range from 4 to 5, it is possible to apply Kirchhoff's laws to the capacitance network (Figure 2) and calculate the voltage relationships.

By using this method, we can obtain the following results:

$$V_1 = V_1 \quad .....(1)$$
$$V_2 = V_1(1 + k) \quad ....(2)$$
$$V_3 = V_1(1 + 3k + k^2) \quad .....(3)$$
$$V_4 = V_1(1 + 6k + 5k^2 + k^3) \quad ....(4)$$
$$V_5 = V_1(1 + 10k + 15k^2 + 7k^3 + k^4) \quad ....(5)$$
$$V = V_1 + V_2 + V_3 + V_4 + V_5 \quad ....(6)$$

As can be seen, the equations are rather cumber some and if a string of 8 units is to be analysed, it has to be done a fresh from first principles as the above mentioned results for the five unit case cannot be easily extended to cover the 8 unit case. If k is assumed to be 0.1, the voltage distribution per unit will be as shown in Figure 3.

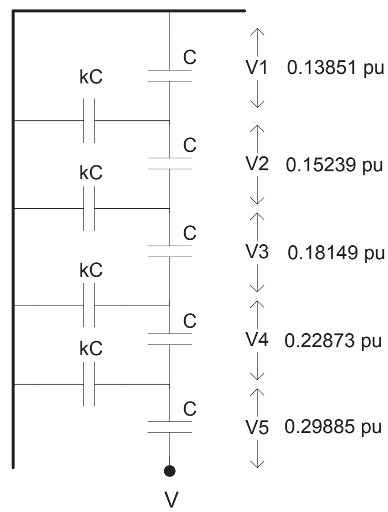


Figure 3 - Voltage distribution along the string

(b) Method of Calculation using Long Line Theory

In this method, we consider the string to have a large number of units up to n and we will use the theory that is used to analyse long transmission lines since the long insulator string can be considered as a long line short circuited at the receiving end, the line conductor end being considered as the sending end.

It can be shown that [2]

$$\frac{V_{m,E}}{V_{n,E}} = \frac{Sinh(m\sqrt{k})}{Sinh(n\sqrt{k})} \quad ..... (7)$$

It is to be noted that $V_{m,E}$ is the voltage at the unit 'm' or at the m[th] unit and that $V_{n,E}$ is the voltage at the n[th] unit which is the line conductor voltage with respect to earth. Here again, k is the ratio of the capacitance between the unit and the tower, to the capacitance of a line unit or self-capacitance.

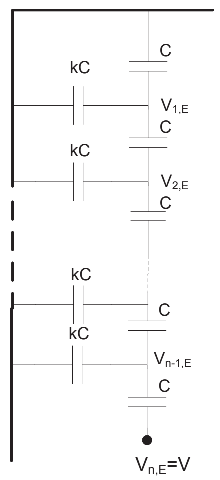


Figure 4 - Configuration for a long insulator string close to the tower

By using this theory for the 5 unit insulator string depicted in Figure 2, we can get the following results:

$$V_{1,E} = V\frac{Sinh(\sqrt{k})}{Sinh(5\sqrt{k})} \quad \text{.... (8)}$$

$$V_{2,E} = V\frac{Sinh(2\sqrt{k})}{Sinh(5\sqrt{k})} \quad \text{.....(9)}$$

$$V_{3,E} = V\frac{Sinh(3\sqrt{k})}{Sinh(5\sqrt{k})} \quad \text{....(10)}$$

$$V_{4,E} = V\frac{Sinh(4\sqrt{k})}{Sinh(5\sqrt{k})} \quad \text{....(11)}$$

$$V_{5,E} = V\frac{Sinh(5\sqrt{k})}{Sinh(5\sqrt{k})} = V \quad \text{....(12)}$$

This method has the advantage in that it can be easily extended for lines comprising different number of units. The only disadvantage of this method is that it contains a hyperbolic function with which many do not like to work. Also, these equations are not applicable when the capacitances between the units and the guard ring have to be considered.

(c) Method of Calculation proposed by the Author (SK Method 1)

The basis of this new method is the use of limited terms in the power series that describes *Sinh(x)*.

$$Sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots \quad \text{......(13)}$$

When this series is used for identifying the voltage, the higher order terms will have less significance and as such it will be possible to truncate the series either at the second term or at the third term. In the first method proposed by the author, the series is truncated at the second term and when this is done, it can be shown that

$$\frac{V_{m,E}}{V_{n,E}} = \frac{m}{n}\left[\frac{1+\frac{1}{6}km^2}{1+\frac{1}{6}kn^2}\right] \quad \text{....(14)}$$

where m can take values from 1 to n.

It is to be noted that $V_{m,E} - V_{(m-1),E} = V_m$;whereV_m is the voltage across the m^th unit.

It can be seen from the above equation that if k=0, i.e., in the absence of capacitance to tower,

$$\frac{V_{m,E}}{V_{n,E}} = \frac{m}{n} \quad \text{...(15)}$$

which is a linear relationship as expected.

The non-linearity arises due to the factor K_A, where

$$K_A = \left(\frac{1+\frac{1}{6}km^2}{1+\frac{1}{6}kn^2}\right) \quad \text{.... (16)}$$

which is a function of k.

Table 1 below gives the values obtained using Kirchhoff's laws for the 5 unit string with k=0.1 and their comparison with values obtained using the long line method and the author's method.

Table 1(a) Voltage distribution comparison (long line theory) (n=5, k=0.1)

Unit No (m)		Using Kirchhoff's Law	Using long line theory	%Error
1	V_1	0.13854	0.13815	+0.2815
2	V_2	0.15239	0.15208	+0.2034
3	V_3	0.18149	0.18134	+0.083
4	V_4	0.22873	0.22890	-0.074
5	V_5	0.29885	0.29953	-0.227

Table 1(b) - Voltage distribution comparison (SK Method 1) (n=5, k=0.1)

Unit No (m)		Using Kirchhoff's Law	Using SK Method 1	%Error
1	V_1	0.13854	0.14353	-3.602
2	V_2	0.15239	0.15823	-3.832
3	V_3	0.18149	0.18529	-2.094
4	V_4	0.22873	0.22824	+0.214
5	V_5	0.29885	0.28471	+4.731

It can be seen that although the error in using SK Method 1 is about 5 %, the results can be accepted because of the simplicity of the method. The error in using the long line theory is less than 0.3 %.

(d) SK Method 2

In this Method 2, in order to obtain a better accuracy, the power series for *Sinh(x)* is truncated at the third term and the following equation is derived:

$$\frac{V_{m,E}}{V_{n,E}} = \frac{m}{n}\left[\frac{1+\frac{1}{6}km^2+\frac{k^2m^4}{120}}{1+\frac{1}{6}kn^2+\frac{k^2n^4}{120}}\right] \quad ...(17)$$

As with the previous case when k=0, i.e., in the absence of capacitance between the unit and the tower, the voltage distribution becomes linear. The non-linearity factor here is K_B, where

$$K_B = \left[\frac{1+\frac{1}{6}km^2+\frac{k^2m^4}{120}}{1+\frac{1}{6}kn^2+\frac{k^2n^4}{120}}\right] \quad ....(18)$$

Table 2 below shows the voltage distribution for a 5 unit insulator string with k=0.1. It can be seen from this table that the error is much lower than when Method 1 is used.

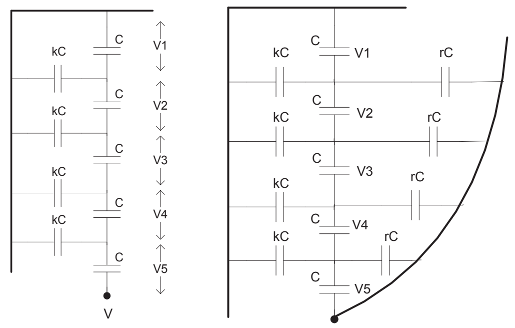
Table 2 - Comparison of voltage distribution along a 5 unit insulator string using SK Method 2 (n=5, k=0.1)

Unit No (m)		Using Kirchhoff's Law	Using SK Method 2	% error
1	V_1	0.13854	0.13845	+0.065
2	V_2	0.15239	0.15241	-0.013
3	V_3	0.18149	0.18168	-0.105
4	V_4	0.22873	0.22901	-0.122
5	V_5	0.29885	0.29845	+0.134

The percentage error in SK Method 2 is in a range similar to that obtained in using the long line theory. The advantage of each of these two methods is that we can obtain with ease, the voltage distribution for a string with any number of units, by substituting values for n and k.

3. A New approach for solving the Voltage Distribution along an Insulator String: *K* Matrix Method

As described earlier, Kirchhoff's laws can be used to calculate the voltage distribution along an insulator string by considering the string as a capacitance network. The method can be used to analyse the network by considering both capacitances, i.e., the capacitance between the unit and the tower and the capacitance between the unit and the guard ring. Figure 5 shows the network for the two cases.



(a) (b)

Figure 5 - (a) Capacitance Network without the guard ring (b) Capacitance network with the guard ring

In this case, 'kC' is the capacitance between the unit and the tower (k<<1.0) and 'rC' is the capacitance between the unit and the guard ring. V_1, V_2, V_3 etc., are the voltages across the relevant insulator units.

Kirchhoff's equations can be written in a well ordered manner in the form of a matrix as shown in Equation (19) for the five unit case using only the self-capacitance and the capacitance between the unit and the tower.

$$\begin{bmatrix} (1+k) & -1 & 0 & 0 & 0 \\ k & (1+k) & -1 & 0 & 0 \\ k & k & (1+k) & -1 & 0 \\ k & k & k & (1+k) & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad ....(19)$$

The new approach described in this paper is based mainly on the proper ordering of the equations and setting them out in a matrix form.

When this method is used, the solution becomes quite simple as shown below.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} (1+k) & -1 & 0 & 0 & 0 \\ k & (1+k) & -1 & 0 & 0 \\ k & k & (1+k) & -1 & 0 \\ k & k & k & (1+k) & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \ldots\ldots(20)$$

The solution involves only the inversion of the matrix for which software is available, e.g., in MATLAB or in other mathematical software. Once the matrix is inverted, the last column of the inverted matrix, will in fact gives the voltage distribution along the insulator string.

The remarkable advantage of this method is that it can be extended to solve the voltage distribution in any string irrespective of the number of units it has. Furthermore, this method can be extended to the case when it becomes necessary to consider the capacitance between the unit and the guard ring. Equation (21) given below is the matrix equation for the five unit case in which the unit self-capacitance is 'C', the capacitance between the unit and the tower is 'kC', and the capacitance between the unit and the guard ring is 'rC.

$$\begin{bmatrix} (1+k) & -(1+r) & -r & -r & -r \\ k & (1+k) & -(1+r) & -r & -r \\ k & k & (1+k) & -(1+r) & -r \\ k & k & k & (1+k) & -(1+r) \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \ldots.(21)$$

The solution is given by,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} (1+k) & -(1+r) & -r & -r & -r \\ k & (1+k) & -(1+r) & -r & -r \\ k & k & (1+k) & -(1+r) & -r \\ k & k & k & (1+k) & -(1+r) \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \ldots(22)$$

For the above equations, '*k*' and '*r*' are constants. That implies that the capacitance between each unit and the tower has the same value and that the capacitance between each unit and the guard ring also has a constant value, which will be different from the first value.

Although we assume that the capacitance between a unit and the tower has the same value for all units, often the capacitance between a unit and the guard ring will vary considerably with the units near the line conductor having a higher capacitance than those near the tower arm. That implies,
$r_1 < r_2 < r_3 < r_4$ in Figure 6.

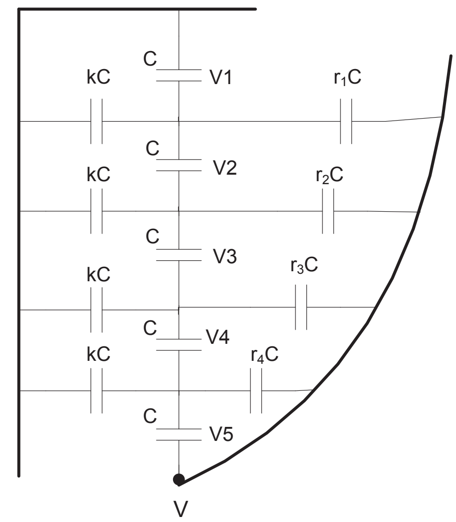


Figure 6 - Capacitance network with varying capacitance to guard ring

In such a case, the method of computation will remain same but the matrix will take a modified form as shown in Equation (23).

$$\begin{bmatrix} (1+k) & -(1+r_1) & -r_1 & -r_1 & -r_1 \\ k & (1+k) & -(1+r_2) & -r_2 & -r_2 \\ k & k & (1+k) & -(1+r_3) & -r_3 \\ k & k & k & (1+k) & -(1+r_4) \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \ldots..(23)$$

The results obtained for three different configurations for a five unit insulator string using the new matrix method are shown in Table 3.

Table 3 - Voltage distribution along a 5 unit insulator string for different values of k and r obtained using the author's new matrix method.

Voltage across each unit	When k=0 r=0	When k=0.1 r=0	When k=0.1 r=0.05	When k=0.1 r=variable
V_1	0.2	0.13854	0.19065	0.15190
V_2	0.2	0.15240	0.16925	0.16179
V_3	0.2	0.18149	0.17324	0.18458
V_4	0.2	0.22873	0.20321	0.22186
V_5	0.2	0.29885	0.26366	0.27990

The value of r will vary as follows:
$r_4 = \frac{k}{2} = 0.05$, $r_3 = \frac{k}{4} = 0.025$, $r_2 = \frac{k}{8} = 0.0125$

$$r_1 = \frac{k}{16} = 0.00625$$

As can be seen above, the matrices are well ordered and can easily be written for an insulator string comprising any number of units.
For example, the matrix equations for an 8 unit insulator string with capacitance between the

unit and the tower and capacitance between the unit and the guard ring will be as given in Equations (24), (25) and (26).

Equation (24) shows the matrix equation for 8 unit insulator string with unit self-capacitance and capacitance between each unit and the tower.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = [K_1]^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \text{.....(24)}$$

where

$$[K_1] =$$

$$\begin{bmatrix} (1+k) & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ k & (1+k) & -1 & 0 & 0 & 0 & 0 & 0 \\ k & k & (1+k) & -1 & 0 & 0 & 0 & 0 \\ k & k & k & (1+k) & -1 & 0 & 0 & 0 \\ k & k & k & k & (1+k) & -1 & 0 & 0 \\ k & k & k & k & k & (1+k) & -1 & 0 \\ k & k & k & k & k & k & (1+k) & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{.....(24a)}$$

Equation (25) shows the matrix equation for a 8 unit insulator string with unit self-capacitance and capacitance between each unit and the tower and capacitance between each unit and the guard ring.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = [K_2]^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \text{.....(25)}$$

where

$$[K_2] =$$

$$\begin{bmatrix} (1+k) & -(1+r) & -r & -r & -r & -r & -r & -r \\ k & (1+k) & -(1+r) & -r & -r & -r & -r & -r \\ k & k & (1+k) & -(1+r) & -r & -r & -r & -r \\ k & k & k & (1+k) & -(1+r) & -r & -r & -r \\ k & k & k & k & (1+k) & -(1+r) & -r & -r \\ k & k & k & k & k & (1+k) & -(1+r) & -r \\ k & k & k & k & k & k & (1+k) & -(1+r) \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{.....(25a)}$$

Equation (26) shows the matrix equation for a 8 unit insulator string with unit self-capacitance and capacitance between each unit and the tower and a varying capacitance between each unit and the guard ring.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = [K_3]^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \text{.....(26)}$$

where

$$[K_3] =$$

$$\begin{bmatrix} (1+k) & -(1+r_1) & -r_1 & -r_1 & -r_1 & -r_1 & -r_1 & -r_1 \\ k & (1+k) & -(1+r_2) & -r_2 & -r_2 & -r_2 & -r_2 & -r_2 \\ k & k & (1+k) & -(1+r_3) & -r_3 & -r_3 & -r_3 & -r_3 \\ k & k & k & (1+k) & -(1+r_4) & -r_4 & -r_4 & -r_4 \\ k & k & k & k & (1+k) & -(1+r_5) & -r_5 & -r_5 \\ k & k & k & k & k & (1+k) & -(1+r_6) & -r_6 \\ k & k & k & k & k & k & (1+k) & -(1+r_7) \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{.....(26a)}$$

In the analysis of long strings, the use of Kirchhoff's equations becomes cumbersome and therefore one will have to resort to the use of the long line theory to obtain a reasonable result. The long line theory used for this analysis unfortunately cannot be applied when the capacitance between the unit and the guard ring has to be considered.

The new K Matrix method developed by the author removes all restrictions and an insulator string of any length and comprising any number of units can be analysed with considerable ease taking into consideration both capacitance between the unit and the tower and the capacitance between the unit and the guard ring.

Theoretically, it is best to operate the insulator string with same voltage across each insulator unit, even though this may not be practically possible. Equations (23) and (26) can be used to determine the conditions that will have to be satisfied in order to achieve the ideal voltage distribution when there is 100 % string efficiency.

In the case of the five unit string, the ideal condition will be achieved when,

$$r_1 = \frac{k}{4}$$
$$r_2 = \frac{2k}{3}$$
$$r_3 = \frac{3k}{2}$$
$$r_4 = 4k \quad \text{.....(27)}$$

Similarly, for the 8 unit string, the ideal condition will be achieved when,

$$r_1 = \frac{k}{7}$$
$$r_2 = \frac{2k}{6}$$
$$r_3 = \frac{3k}{5}$$
$$r_4 = \frac{4k}{4}$$
$$r_5 = \frac{5k}{3}$$
$$r_6 = \frac{6k}{2}$$
$$r_7 = \frac{7k}{1} \qquad \text{….. (28)}$$

By examining Equations 23 and 26, the conditions required can be obtained merely by observation. This will enable us to determine with ease, the requirements for an insulator string of any length for 100% string efficiency.

4. Application of the K Matrix Method for the Most General Configuration.

So far for all configurations presented here, the capacitance between each unit and the tower in an insulator string has been assumed to be the same. Though this appears to be reasonable considering the position of the insulator string and the tower, in the most general case, it will not be necessary to make this assumption. Figure 7 shows the configuration for the most general case of a 5 unit insulator string where $k_1 \neq k_2 \neq k_3 \neq k_4$.

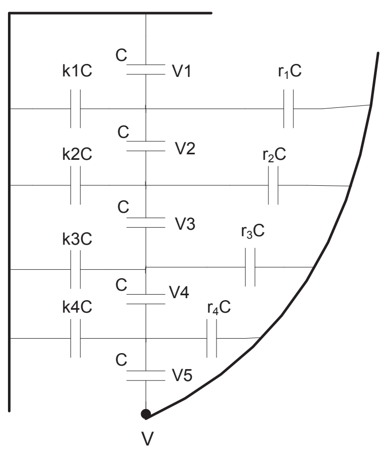


Figure 7 - Configuration for the most general case of the insulator string

Equation (29) is the matrix equation relevant to the configuration given in Figure 7.

$$\begin{bmatrix} (1+k_1) & -(1+r_1) & -r_1 & -r_1 & -r_1 \\ k_2 & (1+k_2) & -(1+r_2) & -r_2 & -r_2 \\ k_3 & k_3 & (1+k_3) & -(1+r_3) & -r_3 \\ k_4 & k_4 & k_4 & (1+k_4) & -(1+r_4) \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \qquad \text{…..(29)}$$

The solution for the voltage distribution along the string is given by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} (1+k_1) & -(1+r_1) & -r_1 & -r_1 & -r_1 \\ k_2 & (1+k_2) & -(1+r_2) & -r_2 & -r_2 \\ k_3 & k_3 & (1+k_3) & -(1+r_3) & -r_3 \\ k_4 & k_4 & k_4 & (1+k_4) & -(1+r_4) \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \qquad \text{.....(30)}$$

This can easily be extended for the 8 unit insulator string which will give

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = [K_4]^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \qquad \text{…..(31)}$$

where

$$[K_4] =$$

$$\begin{bmatrix} (1+k_1) & -(1+r_1) & -r_1 & -r_1 & -r_1 & -r_1 & -r_1 & -r_1 \\ k_2 & (1+k_2) & -(1+r_2) & -r_2 & -r_2 & -r_2 & -r_2 & -r_2 \\ k_3 & k_3 & (1+k_3) & -(1+r_3) & -r_3 & -r_3 & -r_3 & -r_3 \\ k_4 & k_4 & k_4 & (1+k_4) & -(1+r_4) & -r_4 & -r_4 & -r_4 \\ k_5 & k_5 & k_5 & k_5 & (1+k_5) & -(1+r_5) & -r_5 & -r_5 \\ k_6 & k_6 & k_6 & k_6 & k_6 & (1+k_6) & -(1+r_6) & -r_6 \\ k_7 & k_7 & k_7 & k_7 & k_7 & k_7 & (1+k_7) & -(1+r_7) \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad \text{…. (32)}$$

From the above equations, the conditions necessary for the insulator string to have 100% string efficiency can be derived.

The conditions for the 5 unit string to have 100 % string efficiency are as follows:

$$r_1 = \frac{k_1}{4}$$
$$r_2 = \frac{2k_2}{3}$$
$$r_3 = \frac{3k_3}{2}$$
$$r_4 = 4k_4$$

And for the 8 unit string

$$r_1 = \frac{k_1}{7}$$
$$r_2 = \frac{2k_2}{6}$$
$$r_3 = \frac{3k_3}{5}$$
$$r_4 = \frac{4k_4}{4}$$
$$r_5 = \frac{5k_5}{3}$$
$$r_6 = \frac{6k_6}{2}$$
$$r_7 = \frac{7k_7}{1}$$

Though these give the ideal conditions for 100 % string efficiency, in practice due to practical difficulties it will never be possible to satisfy these conditions.

5. Application of the K Matrix Method for the Most Simple Configuration

The most simple configuration for the insulator string is for the string to be remote from the tower and unaffected by the guard ring. This configuration is the one depicted in Figure 1 for the five unit insulator string. Equation (33) is the matrix equation for this configuration.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \text{.....(33)}$$

The solution for the voltage distribution along the string is given by Equation (34).

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \text{...(34)}$$

As is the case with previous cases, the matrix equation for the 8 unit insulator string for the most simple case is,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = [K_0]^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V \end{bmatrix} \quad \text{...(35a)}$$

where

$$[K_0] = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{...(35b)}$$

It is seen that, the solution obtained by using Equations (34) and (35), satisfies the condition that for the most simple case, the voltage distribution along the string is uniform irrespective of the number of units.

6. Conclusion

Every electrical power engineer should be aware of the methods available for the calculation of voltage distribution along an insulator string. The direct use of Kirchhoff's equations is simple enough in the case of short strings comprising 3-4 insulator units and will become rather complicated for strings with eight or more units.

For long strings, the theory based on long line analysis can be applied to obtain reasonable results. However, this method can be used only when the self-capacitance of units and the capacitance between them and the tower have to be considered. It will not be applicable when the capacitance to the guard ring also has to be considered.

The new K Matrix Method developed by the author can easily be used to analyse short as well as long strings when both capacitance between the units and the tower and capacitance between the units and the guard ring are present. Furthermore, it accommodates the case where the capacitances between the units and the tower become variable. The method is simple and can be used in classroom teaching to electrical engineering students as well.

In the first part of the paper, the author has also developed two simple equations to enable the calculation of the voltage distribution along any insulator string (short or long) with ease and with a fair degree of accuracy. These equations will be helpful to all students of electrical power engineering.

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