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A Design Nomogram for a Horizontally-Fractured Geothermal Reservoir to determine the Production Temperature

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Abstract: The paper presents a graphical technique for the rapid analysis of the complex, non-linear equation that describes the variation of the production temperature of a horizontally-fractured geothermal well. The nomogram presented for the evaluation of the production temperature incorporates the mass flow rate, fracture width, fracture length, number of conductive fractures, host rock temperature, and the production time. The range of the fracture length considered in the nomogram varies from 100 m to 1000 m whereas host rock temperature can be selected from a range of 100 ^{o}C to 260 ^{o}C. The production temperature predicted by the nomogram agrees well with the analytical solution. The most attractive feature of the proposed technique is its extreme simplicity and speed of operation.

Keywords: nomogram, PyNomo software, geothermal wells, production temperature.

1. Introduction

Among the renewable energy options, geothermal energy has the potential to contribute significantly towards alleviating the world's energy-climate predicament by providing baseload power (Tester et al., 2007). In order for geothermal power generation to become a major energy source, the development in non-volcanic are as becomes essential (IEA, 2011). The Enhanced Geothermal Systems(EGS) is a universal technology(Figure 1) that can be applied in non-volcanic areas, and EGS circulates the water between the injection and the production wells, extracting the stored geothermal energy in the reservoir at depths of 3-5 km. Forced fluid flow occurs both in natural and artificial fracture systems, and heat is extracted by convective heat transfer. One of the main challenges is ensuring that the reservoir has sufficient permeability for the process to function appropriately, so hydraulic stimulation is used initially to enhance the permeability of the host rock.

Both analytical and numerical studies have been conducted to determine the coupled thermal, mechanical, and hydraulic behavior of fractured systems (Tsang,1991). Although numerical models offer the advantage of enabling the consideration of complex geometry and boundary conditions, analytical studies are important because they provide a basis for the use of numerical models. Analytical solutions in geothermal engineering incorporate the mechanism of forced, convective heat flow in the fractures. The analytical solutions presented by Gringarten et al.(1975) were analyzed in parallel with equidistant vertical fractures. The solution can be easily converted in to horizontal fractures in a thermal isotropic media using simple arithmetic. In fact, their equivalent-temperature approach can be incorporated to highlight the importance of the multi-fracture concept in the extraction of geothermal energy. The analytical model developed by Yang and Yeh (2011) can be used to obtain the geothermal production temperature. Their approach involved numerical inversion of the Laplace transform to determine the fluid temperature. The importance of the spacing of wells, the radii of the wells, the thickness of the reservoir, and the flow rates in a multi-well system was illustrated. A more general form of a solution for forced convection and conduction was presented by Carslaw and Jaeger(1959) and Bodvarsson(1969). As illustrated in Bodvarsson's solution, several variables affect the final production temperature, including the fracture length, fracture width, mass flow rate, number of conductive fractures, and the time period for production. The contribution of each factor to the final production temperature is difficult to visualize because the final form of solution is not solved by straightforward, manual calculations.

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In this article, we present a special graphic and nomographic solution for the problem.

Nomography (Otto, 1963) is one approach that can be used to construct graphical representations of mathematical relationships. Nomography was invented in the late nineteenth century (Evesham, 1982). During the first half of the twentieth century, it was used extensively until personal computers and calculators became a more common way to make rapid calculations. However, still there are cases for which this ′lost art′ is invaluable. Nomograms can be used effectively to conduct sensitivity analyses or to develop an equation and indicate the relationships between its underlying variables. The recent examples of nomograms can be found in many fields, including the medical field (Marasco et al., 2011;Kattan and Marasco, 2010; Fagan, 1975). The approach provides a simple graphical method that can be used to study the effect of each parameter and the effects of various combinations of different parameters on the final solution. In the present work, we developed a nomogram for fluid-carrying, horizontal fractures characterized by convection and conduction-coupled systems. The analytical solution provided by Bodvarsson was used to develop the production temperature by combining the other variables.

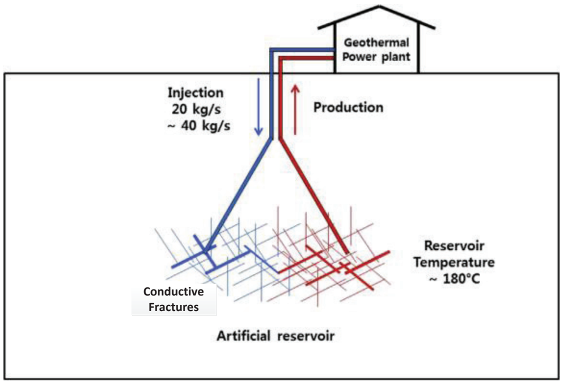


Figure 1 - Conceptual diagram showing a typical enhance geothermal system (EGS)

1.1 PyNomo Software

PyNomo software was used in this study to construct the nomogram. PyNomo is an open-source software package, written in Python, that can be used to draw precision nomograms. The output is in vector form in a PDF or EPS file. PyNomo directly supports nine canonical forms of nomograms (Allcock and Jones, 1950;Levens, 1959; Doerfler, 2009) based simply on the format of the equation, so there is no need to convert the equation to the standard nomographic determinant or to use geometric relationships. PyNomo also supports more complicated equations that are in general in the determinant form, so it can produce an output for any equation that can be plotted as a nomogram.

2. A Nomogram to determine the production temperature

The nomogram for the determination of production temperatures was constructed by assuming that the thermodynamic properties of rock and water were constant. These standard thermodynamic properties are given in Table 1. The original form of Bodvarsson equation which has been derived for a host rock temperature of 0^{o}C is given in Eq.1.

$$T_{ext} = \Delta T_o erfc\left[\frac{2k}{C_w q} \mathrm{X} \frac{x}{2\sqrt{\alpha t}}\right] \quad (1)$$

where x is the distance from the injection well, t is the time that has passed after the injection of water, ΔT_o is the initial temperature difference between the injected fluid and the rock,q is the mass flow rate, k is the thermal conductivity of the rock, α is the thermal diffusivity of the rock, *erfc* and inverfs are the error function and the inverse of the error function.

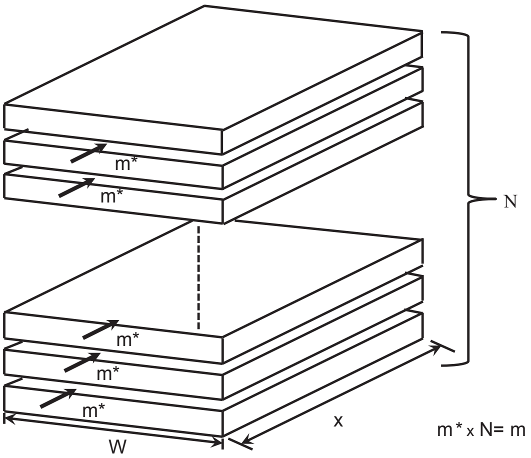
In order to simplify Eq.1, the mass flow rate term is substituted by the product of aperture (h), velocity(v) and the density of water (ρ_f). In the second stage, the velocity component (v) in the Bodvarsson equation was expressed in terms of mass flow rate(m), fracture aperture (h), number of conductive fractures (N), fracture width (W), and density (ρ).This approach allowed us to eliminate the fracture aperture in the final form of the nomogram. This yields the original Bodvarsson solution to the modified form given in Eq.(2):

$$T_{ext} = T_{rock} - \Delta T_o erfc\left[\frac{2k}{C_w \frac{m}{NW}} \mathrm{X} \frac{x}{2\sqrt{\alpha t}}\right] \quad (2)$$

where x is the distance from the injection well,t is the time that has passed after the injection of water, $-\Delta T_o$ is the initial temperature difference between the injected fluid and the rock, h is the fracture aperture, vi s the flow velocity, k is the thermal conductivity of the rock, α is the thermal diffusivity of the rock, ρ_f is density of the water, c_w is the specific thermal capacity of water, N and W are the number of conductive fractures and fracture width respectively, and m is the mass flow rate. Figure 2 illustrates the different geometrical variables together with the flow rate.

Table 1-Standard thermodynamic properties of rock and water(Similar to igneous type rock)

Property	Value
Specific heat capacity (rock)	2.75 W/m.K
Thermal conductivity (rock)	915 J/kg.K
Density (rock)	2600 kg/m^3
Specific heat capacity (water)	4178 J/kg.K
Thermal conductivity (water)	0.58 W/m.K
Density (water)	1000 kg/m^3



(not to scale)

Figure 2 - Illustration of different geometrical variables and the flow rate in Bodvarsson's equation

It is understood that the key variables that affect the final production temperature are the initial rock temperature, mass flow rate, fracture width, fracture length, number of conductive fractures, and the operation time, assuming that the thermal properties of the rock and the injected water remain constant. After rearranging some of the terms in Eq.(2), the final form of the equation used to construct the nomogram was formed by taking the *log* of Eq. (3), which produced a sum of the separate variables on the right-hand side of the equation, which is a form that is suitable for constructing a nomogram.

$$inverfc\left(\frac{T_{ext} - T_{rock}}{\Delta T_o}\right) = \left[\frac{2k}{C_w \frac{m}{NW}} \mathrm{X} \frac{x}{2\sqrt{\alpha t}}\right] \qquad (3)$$

The form of the nomogram was the combination of type 5 (contour plot) and type 3 (N-parallel lines) forms of nomograms, as defined by Doerfler (2009).

Figure 2 shows the nomogram that was developed for determining the production temperature. As shown in Figure 3, the mass flow rate can be varied from 10 kg/s to 60 kg/s, and the fracture width from 10 m to 1000 m. The range of the fracture length goes from 100 m to 1000 m, and the range of the number of conductive fractures from 10 to 60. The production period can be set from 1 to 30 years. Five different contour lines shown in the production temperature contour curves illustrate five different rock temperatures(260, 240, 220, 200, 180^{o}C). Numbers that appear in the production temperature curves 200, 180,160,140,120 denote the initial temperature differences between the inlet fluid(60^{o}C) and the rock temperature.

The isopleth curve illustrates the use of the nomogram for one possible case to determine the production temperature. Reading the nomogram starts with the given mass flow rate (e.g., 30 kg/s) and the fracture width (e.g., 60 m). By drawing a straight line connecting these two values, users can determine the point in index line one(Ref 1). The point in index line two(Ref 2) is determined by connecting the point in index line one(Ref 1) and the fracture length (400 m) with a straight line. The point in index line 3(Ref 3) is determined by connecting the number of fractures (20) and the point in index line 2(Ref 2). The point in line B is determined by connecting the production temperature time and the point in the index line 3. Then, the user must draw a straight line perpendicular to line B until the appropriate rock-temperature contour is intersected. The final production temperature can be determined by drawing a projection line to the production temperature line. If the appropriate host rock temperature is 180^{o}C the relevant production temperature will be 127^{o}C.

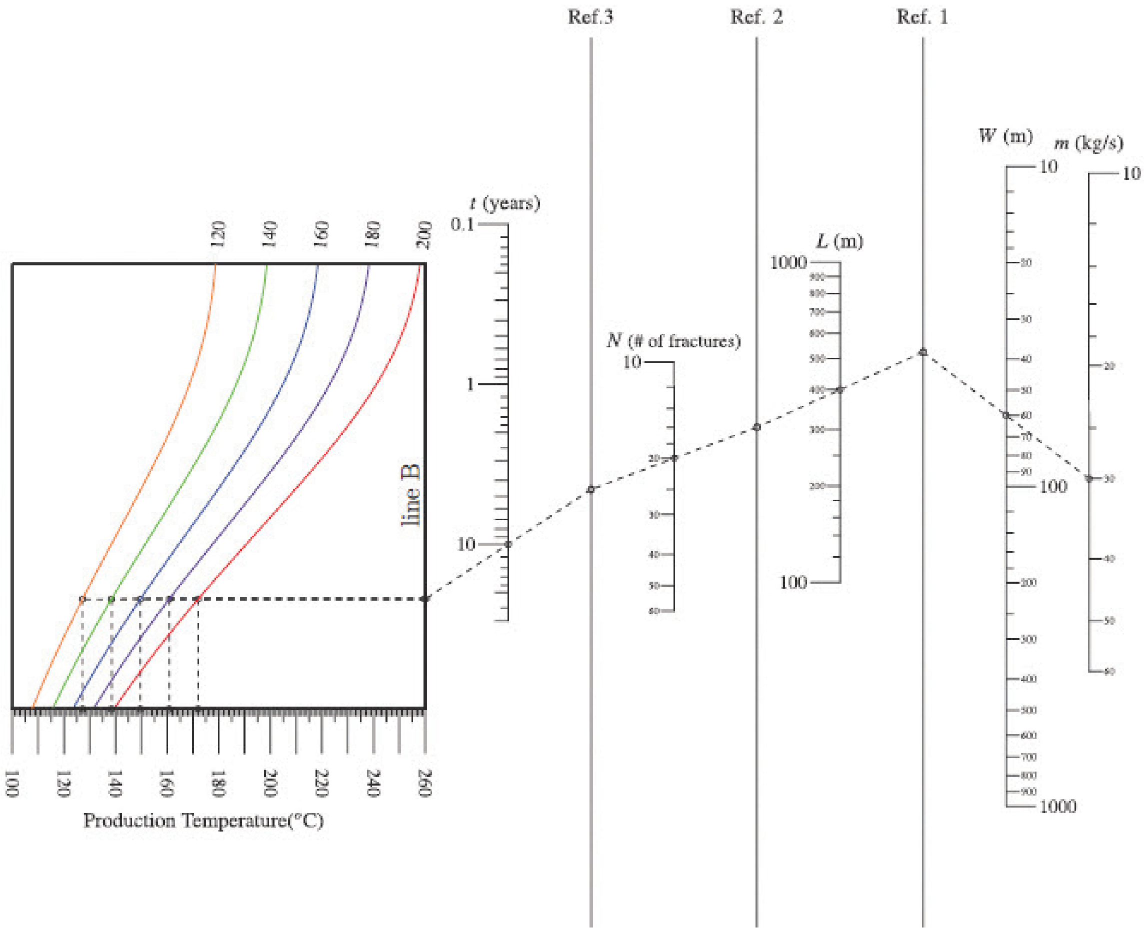


Figure 3- Nomogram for analyzing convection/conductive, coupled geothermal wells (The selected example shows the production temperature after 10 years)

3. Verification of the present work

To verify the Bodvarsson's solution, it was compared with the values obtained by the finite-element method for a convective and conductive, numerical heat transfer model. In this simulation, we used a mass flow rate of 40 kg/s, a fracture aperture of 0.01 m, a fracture length of 600 m, and a fracture width of 100 m. From these values, we determined that the basic flow velocity was 0.04 m/s. Table 2 provides the values that we used for the remaining thermodynamic properties. The production temperatures derived by the Bodvarsson solution and by the finite-element method at different times are compared in Figure 4 and they reach a good agreement.

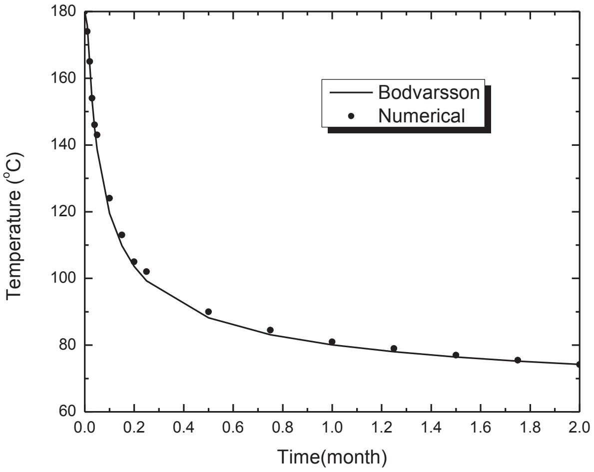


Figure 4 -Comparison of the Bodvarsson solution and a numerical solution

The verification of a nomogram is a key component in its development. To verify our nomographic results, we compared them with analytical solutions provided by a MATLAB program. Several MATLAB calculations with different combinations of variables were conducted to determine the production temperatures at different times. The results of our verification calculations are summarized in Table 2. The error percentages in the production temperatures of nomographic values were computed by comparing with the production temperatures derived by the MATLAB. Some combinations of variables were outside the values predicted by the nomogram. The error percentages associated with these combinations of variables are denoted as 'out of the range' in the Table.

The agreement between the MATLAB results(Eq. (2)) and the results derived from the nomogram was excellent; the maximum error was less than 1%. Similar types of error calculations were made using the same combinations of variables as in Table 1, but with different fracture lengths, i.e., 500 m, 600 m, and 1000 m. We performed the same error calculation for 1500 cases, and the maximum error was always less than 3%.

In principle, an analytical equation can be used to compute the production temperature at a given time, but solving Bodvarsson's equation is not easy without the aid of computers. So, it would also be useful if the production temperatures could be calculated using a simpler approach, such as a nomogram, without elaborate analytical or numerical calculations.

In designing geothermal wells, it would be of interest to determine the limits of different variables, such as the fracture length, fracture width, and mass flow rate, for a required production temperature. Our nomographic solution can also be used easily to determine the limits of different variables without having to solve complex equations several times. As a possible case, the limits of different variables could be determined if the required production temperature after 30 years is 195 ^{o}C with the host rock temperature at 260 ^{o}C. To analyze this particular case, starting from 195 ^{o}C, a line was drawn parallel to the guide lines (isopleth) until it intersected the correct host-rock temperature curve. From the intersection point, the projection line was made to *line B*. The point in *line B* and the appropriate point on the time axis (30 years) were connected, yielding the point in Ref axis 3. This point can be used as the limiting value for the selection of other variables. Another possible advantage of a nomogram is that it provides a simple, graphical tool that can be used to study the effects of different variables on the production temperature. Since our nomogram covers most possible ranges, the selection of different variables can be made easily by drawing a set of different lines parallel to the isopleths. This helps designers select the most appropriate and economical parameters for a given geothermal reservoir.

As shown in the validation tables, some combinations of variables lie outside the range of the nomogram, especially after 20 years of production with a flow rate of 50 kg/s. Although this may seem to limit the usefulness of the nomogram approach, this is not true in practice because such cases represent greater temperature drops after the production period, and they do not warrant further interest or investigation because their costs would be prohibitive. Therefore, as illustrated, the range of production temperatures predicted by the nomogram probably covers most cases of practical and economic interest.

4. Conclusions

The Bodvarsson solution, which provides the basis for the nomogram, was first verified by a forced convection/conduction numerical model. We designed and constructed a nomogram for determining the production temperature for a forced-convection/conduction, coupled system, incorporating the mass flow rate, fracture width, fracture length, number of conductive fractures, and the production time. The proposed nomogram was verified against Bodvarsson's solution using several possible combinations of variables. The nomogram developed in this study can handle a large number of possible combinations of different variables that affect the production temperature.

Table 2 - Comparison of nomogram results with MATLAB calculations for a fracture length of 400 m for three different time periods, i.e., 5, 10, and 20 years with two different conductive fractures(*N*) 20 and 30.

			Fracture width (m)							
			40		60		80		100	
Flow rate (kg/s)	Rock temperature (ºC).	Years	Error%		Error%		Error%		Error%	
			N-20	N-30	N-20	N-30	N-20	N-30	N-20	N-30
30	260	5	0.41	0.68	0.71	0.14	0.14	0.12	0.25	0.54
		10	Out	0.82	0.55	0.38	0.57	0.28	0.46	0.19
		20	out	0.79	0.81	0.56	0.14	0.70	0.55	0.34
	240	5	0.78	0.42	0.42	0.10	0.14	0.14	0.07	0.54
		10	out	0.29	0.29	0.38	0.50	0.04	0.45	0.13
		20	out	0.25	0.26	0.36	0.14	0.38	0.36	0.53
	220	5	0.52	0.66	0.66	0.04	0.13	0.02	0.33	0.53
		10	out	0.35	0.35	0.36	0.41	0.27	0.42	0.08
		20	out	0.42	0.97	0.78	0.52	0.65	0.77	0.32
	200	5	0.21	0.32	0.32	0.01	0.69	0.31	0.64	0.50
		10	out	0.42	0.42	0.34	0.31	0.02	0.41	0.01
		20	out	0.61	0.42	0.34	0.31	0.02	0.41	0.01
	180	5	0.15	0.59	0.59	0.11	0.12	0.12	0.13	0.48
		10	out	0.58	0.51	0.32	0.20	0.24	0.39	0.07
		20	out	0.10	0.82	0.30	0.66	0.59	0.31	0.29
40	260	5	0.11	0.56	0.56	0.59	0.66	0.15	0.34	0.10
		10	out	0.00	0.47	0.81	0.82	0.39	0.19	0.30
		20	out	out	out	0.70	0.82	0.56	0.81	0.04
	240	5	0.26	0.36	0.35	0.38	0.42	0.01	0.10	0.05
		10	out	0.23	0.23	0.36	0.28	0.37	0.30	0.33
		20	out	out	out	0.80	0.25	0.36	0.85	0.77
	220	5	0.43	0.78	0.78	0.66	0.65	0.04	0.32	0.00
		10	out	0.74	0.74	0.49	0.32	0.36	0.42	0.32
		20	out	out	out	0.18	0.42	0.78	0.89	0.67
	200	5	0.25	0.00	0.01	0.41	0.32	0.03	0.02	0.58
		10	out	0.49	0.49	0.63	0.42	0.34	0.56	0.31
		20	out	out	out	0.27	0.61	0.56	0.16	0.56
	180	5	0.10	0.30	0.30	0.10	0.59	0.11	0.29	0.14
		10	out	0.20	0.20	0.06	0.51	0.32	0.01	0.29
		20	out	out	out	0.38	0.82	0.30	0.15	0.42
50	260	5	out	0.48	0.48	0.84	0.51	0.51	0.71	0.57
		10	out	out	out	0.47	0.38	0.69	0.82	0.98
		20	out	out	out	out	out	0.49	0.82	0.56
	240	5	out	0.26	0.24	0.53	0.66	0.22	0.42	0.09
		10	out	out	out	0.58	0.51	0.61	0.29	0.37
		20	out	out	out	out	out	0.26	0.25	0.35
	220	5	out	0.72	0.72	0.18	0.22	0.27	0.66	0.46
		10	out	out	out	0.71	0.65	0.52	0.35	0.36
		20	out	out	out	out	out	0.73	1.00	0.78
	200	5	out	0.45	0.49	0.42	0.38	0.32	0.32	0.03
		10	out	out	out	0.86	0.02	0.41	0.43	0.34
		20	out	out	out	out	out	0.49	0.61	0.56
	180	5	out	0.21	0.21	0.69	0.56	0.40	0.45	0.13
		10	out	out	out	0.21	0.14	0.28	0.51	0.32
		30	out	out	out	out	out	0.20	0.10	0.30

The nomogram can be used easily to study the effects of different variables on the final production temperature. This nomogram would be useful in conducting preliminary sensitivity studies before complex, numerical simulations of geothermal wells are undertaken.

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