ENGINEER - Vol. XXXXI, No. 01, pp. 07-12, 2008

Bed Shear Stress in Unsteady Open Channel Flow Over Rough Beds

K.P.P. Pathirana, P.C. Ranasinghe and U.R. Ratnayake

Abstract: Very few studies have been carried out in the past in estimating bed shear stress in unsteady flows over rough channel beds. The shear velocity derived from de Saint Venant equations was compared with the steady state formulation using experimental data. The difference in shear velocities predicted by steady and unsteady formulations increases with unsteadiness of the flow. In addition, the shear velocity in accelerating flows are generally higher than that in decelerating flows and this information is very useful for sediment transport studies in unsteady flows. The error in computing shear velocity in unsteady flows using steady state formula was also quantified.

Keywords: Bed shear stress, Unsteady flow, Shear velocity

1. Introduction

Bed shear stress is one of the important parameters required in estimating sediment transport rates in open channels, rivers and streams. The computation of bed shear stress in steady, uniform flow has been extensively and rather conclusively investigated by many researchers. However, the flow in rivers and streams is mostly unsteady and non-uniform. The computation of bed shear stress even in this type of flows is still based on the formulations that are developed for steady, uniform flow conditions.

Very few studies have been reported in the past investigating the bed shear stress in unsteady flows and most of these investigations do not provide any conclusive results. Cardoso et. al, [1] studied the structure of spatially accelerating flows in a smooth open channels based on laboratory experiments. The influences of acceleration on the velocity profiles, turbulent intensity profiles and longitudinal evolution of bottom shear stress were investigated. A study on the friction coefficient in unsteady open channel flow over gravel beds was presented by Tu and Graf[11]. The present study is aimed at investigating the computation of bed shear stress in unsteady open channel flow over rough bed using extensive laboratory experiments.

2. Friction Velocity in Unsteady Flow

The unsteady flow in open channels is expressed by the following partial differential equations developed by *de Saint Venant* (Henderson, [5]).

$$\frac{dA}{\partial t}+\frac{\partial(AU)}{\partial x}=0 \quad (1)$$

$$-\frac{\partial(y+z_0)}{dx}=-\frac{1}{g}\frac{\partial U}{dt}+\frac{U}{g}\frac{\partial U}{dx}+\frac{1}{y}\frac{\tau_0}{pg} \quad (2)$$

where, A is the cross-sectional area, U the depth-averaged velocity, y the water depth, p the water density, z_0 the elevation of channel bottom slope, x the longitudinal coordinate, t the time and τ_0 the bed shear stress which is expressed as $\tau_0 = \rho u_*^2$. The friction velocity or shear velocity (u_*) can be obtained from Eq.(2) as;

$$u_* = \sqrt{gy\left[S-\frac{\partial y}{dx}-\frac{U}{g}\frac{\partial U}{dx}-\frac{1}{g}\frac{\partial U}{dt}\right]} \quad (3)$$

Where, S = channel bottom slope $=-\partial z_0/\partial x$.The derivatives dy/dx and $\partial U/\partial x$ in Eq.(3) are changed to time derivatives $\partial y/\partial t$ and $\partial U/\partial t$ using the wave velocity concept (Tu and Graf, [11]), which is described below;

For an observer standing on the wave crest and moving along with the wave velocity, C_r for a constant discharge, the unit discharge, $q(x,t)$ = yU, is constant and the total variation of q is zero.

$$dq=\frac{\partial q}{dt}dt+\frac{\partial q}{dx}dx=0 \quad (4)$$

Eng. (Dr.) K.P.P. Pathirana, *B.Sc. Eng. (Hons.)(Peradeniya)M.Eng., Ph.D., C.Eng., FIE(SL), MICE(London), Senior Lecturer, Department of Civil Engineering, University of Peradeniya.*

Eng. P.C. Ranasinghe, *B.Sc. Eng. (Hons.), (Peradeniya), AMIE(SL), Civil Engineer, Road Development Authority.*

Eng. (Dr.) U.R. Ratnayake, *B.Sc. Eng. (Hons). (Peradeniya), M.Eng., D.Eng. (AIT, Bangkok), C.Eng., MIE(SL), Senior Lecturer, Department of Civil Engineering, University of Peradeniya.*

Consequently, the wave velocity C can be expressed as;

$$C_q = \frac{dx}{dt} = -\frac{\partial q}{dt} \div \frac{\partial q}{dx} \quad (5)$$

The continuity equation (Eq.(1)) can be written as;

$$\frac{\partial y}{\partial t} + \frac{\partial q}{dx} = 0 \quad (6)$$

From Eqs.(5) and (6);

$$C_q = \frac{\partial q}{dt} \div \frac{\partial y}{\partial t} = \left[U\frac{dy}{dt} + y\frac{dU}{dt}\right] \div \frac{dy}{dt} \quad (7)$$

$$= U + y\left[\frac{dU}{dt} \div \frac{dy}{dt}\right]$$

Since $q = yU$; $\quad \frac{\partial q}{dx} = y\frac{\partial U}{dx} + U\frac{\partial y}{dx} \quad (8)$

Substituting Eqs. (6) and (8) in Eq.(7), the wave velocity can be expressed as;

$$C_q = -\frac{y\frac{dU}{dt} + U\frac{\partial y}{\partial t}}{y\frac{dU}{\partial x} + U\frac{\partial y}{\partial x}} \quad (9)$$

Therefore, for a constant velocity, the wave velocity is;

$$C_y = \frac{\partial x}{dt} = -\left[\frac{dy}{dt} \div \frac{dy}{dx}\right] \quad (10)$$

and for a constant water depth, the wave velocity is;

$$C_u = \frac{dx}{dt} = -\left[\frac{dU}{dt} \div \frac{dU}{dx}\right] \quad (11)$$

and consequently,

$$\frac{\partial y}{dx} = -\frac{1}{C_y}\frac{\partial y}{dt} \quad \text{and} \quad \frac{\partial U}{dx} = -\frac{1}{C_u}\frac{\partial U}{dt} \quad (12)$$

Considering the kinematic wave, for which, velocity is a function of water depth only i.e., *U=f(y)*,it can be assumed that the wave velocity $C = C_y = C_u$.

By substituting Eq.(12) in Eq,(3), the shear velocity for unsteady flow is obtained as;

$$u_{*un} = \sqrt{\left[gy\left(\frac{1}{C}\frac{\partial y}{\partial t} + S\right)\right] - \left[y\frac{\partial U}{\partial t}\left(1 - \frac{U}{C}\right)\right]} \quad (13)$$

Where, $C = U + \sqrt{gy}$.

Based on the experimental investigations carried out for shear velocity computations on rough bed due to unsteady flow conditions, Kabir [6] concluded that the second term of Eq.(13) which includes *dU/dt* term, had almost negligible effect on u_* values. He also showed that shear velocity is more sensitive to water level variation than to mean velocity variation. Therefore, friction velocity in unsteady flows can be expressed by the following simplified equation,

$$u_{*un} = \sqrt{\left[gyS + \frac{1}{C}gy\frac{\partial y}{\partial t}\right]} \quad (14)$$

The shear velocity in steady flow, u_{*s} is computed using Eq. (15).

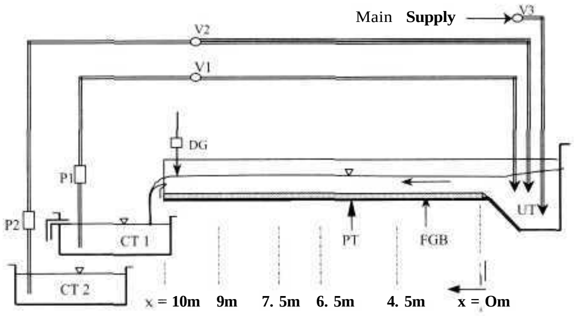
$$u_{*s} = \sqrt{gyS} \quad (15)$$

2. Experimental Set-Up and Procedure

The experiments were carried out in a 10 m long, 0.4 m wide, 0.5 m deep rectangular, re-circulating, tilting flume in the Hydraulics Laboratory of Faculty of Engineering, University of Peradeniya. A schematic diagram of the experimental set-up is shown in Figure 1.

The upstream tank is 1.5 m long, 1 m deep and 1 m wide and it is shaped so that - types is gradually in width end to fit with the channel entrance, at the end in order to avoid any undesired sudden undulations of the flow. A sharp crested weir is fixed at the end of the channel with a depth gauge to facilitate the estimation of flow discharges. Two pressure transducers were used to measure the flow height variation when different hydrographs are passed over the channel bed.

Natural river sand was sieved and the grains that passed through the 8 mm sieve but which were retained on the 2.3 mm sieve were used for the experiments. The mean diameter of sediment was 4 mm. The fixed bed was prepared by gluing a single layer of sediments on top of a plywood sheet with cement slurry and this sheet was fixed on to the channel bed.



UT = Upstream tank CTl & CT2 = Collecting tank
P1 & P2 = Pumps FGB = Fixed gravel bed
V1 & V2 = Valves DG = Depth gauge
PT = Pressure transducers

Figure 1: Experimental set-up

Unsteady flow was established in the channel by passing a hydrograph over fixed rough bed, which was done by adding a wave of period of DT and height y_p, to a base flow y_b. Thus, the base flow of a known quantity was first made over the fixed bed using the set-up used for the steady flow runs. Using an additional pump in excess to those used in the re-circulating system and with preset valve opening and timing the pump running durations, six hydrographs were passed over the steady base flow. All these hydrographs were repeated several times to ensure repeatability.

The experiments were repeated for four different channel slopes and four different base flow discharges, each containing six hydrographs, to have a total number of 96 test runs. The water depths were measured continuously during the tests using the pressure sensors and all observations were logged into the computer using A/D converter. The range of flow parameters used in the experimental runs is summarized in Table 1.

3. Results and Discussion

A total of 96 hydrographs was investigated during the experimental runs. Unsteadiness of test cases was quantified by using a hydrograph parameter (HYDP), which is defined as (Kabir, [6]);

$$HYDP = \frac{2(y_p - y_b)y_p}{(u_{*b}DT)^2} \qquad \text{..........(16)}$$

where y_b is the water depth in base flow, y_p the water depth at peak, u_{*b} the shear velocity in base flow and DT the duration of hydrograph. The HYDP of test cases varies between $3.17*10^{-4}$ (most unsteady one) and $1.52*10^{-5}$ (least unsteady one). The shear velocities were computed using two methods; one with Eq.(14) which was specifically derived for unsteady flows using de Saint Venant equations and the other one with Eq.(15), used for steady flow runs. As a typical result of the unsteady flow runs, the time variation of water depth and shear velocities during the passage of a hydrograph is shown in Figure 2.

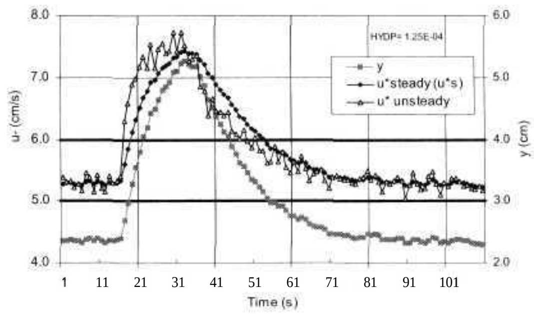


Figure 2: Variation of water depth and shear velocity in an unsteady flow run.

The shear velocities estimated from the two methods seem to be close to each other in base flow as the state of the flow may be still steady. However, when passing the rising limb of the hydrograph, the shear velocities based on unsteady flow equations were larger than that from the steady state formula and this behaviour was reversed when passing the falling limb. It can also be noted that, for the

Table 1: The range of hydraulic parameters used in the experimental runs

Slope (S)	No of test runs	Base flow (Q) (l/s)	Water depth in base flow (y_b) (cm)	Water depth at peak (y_p) (cm)	Time to reach peak water depth (Tp) (sec)	Duration of hydrograph (DT) (sec)
0.0135	24	3.3 - 23.2	2.4 - 8.1	3.80 - 8.75	9 - 31	57 - 107
0.0110	24	3.3 - 23.2	2.5 - 8.7	3.98 - 10.27	10 - 32	61 - 144
0.0080	24	3.3 - 23.2	3.0-9.6	4.29 - 11.28	10 - 33	58 - 106
0.0055	24	3.3 - 23.2	3.1 - 10.3	4.52 - 12.32	11 -32	57 - 108

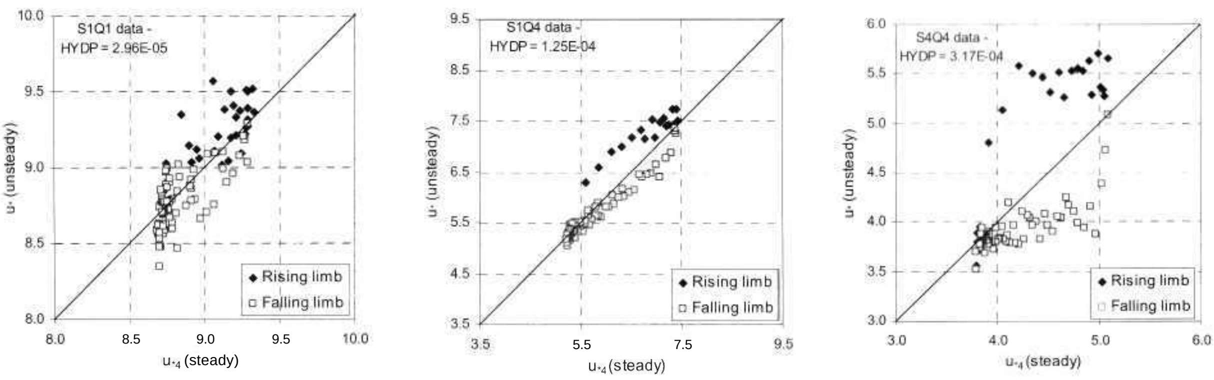


Figure 3: Comparison of shear velocities computed by steady and unsteady state formula during the passage of a hydrograph

[(a) HYDP = $2.96*10^{-5}$ (b) HYDP = $1.25*10^{-4}$ and (c) HYDP = $3.17*10^{-4}$]

same water depth in a hydrograph, the actual shear velocity and thus, the bed shear stress is larger in the rising branch than that in the falling branch. This phenomenon is very important in quantifying sediment transport in unsteady flows.

Figure 3 compares the u* values based on the steady and unsteady formula during the passage of three different hydrographs having HYDP of $2.96*10^{-5}$, $1.25*10^{-4}$ and $3.17*10^{-4}$ which are in the order of increasing unsteadiness.

The prediction of shear velocity from both the steady and unsteady flow equations appeared to be very close to each other if unsteadiness of the flow is low as shown in Figure 12(a). When unsteady nature of the flow increases the difference in shear velocities derived from the two methods seems to increase significantly for the rising and falling limbs of the hydrographs as illustrated in Figures 3(b) and 3(c). These plots clearly show that the shear velocities in the rising limb are higher than those in the falling limb.

During the passage of the hydrographs, the shear velocity obtained from the unsteady flow equation always reaches its maximum value a little before the water level reaches its maximum, as shown in Figure 2. This has also been observed in previous studies carried out in unsteady open channel flows over rough beds (Kabir [6] and Graf et al. [4]). At the peak of the hydrograph, the shear velocities given by both the unsteady and steady state formulas are almost the same. This is due to the presence of the gradient term *dy/dt* in unsteady flow equation, as the term becomes zero at the water level peak.

The relative time lag t_L/DT (where t_L is lag time between the peak shear velocity from unsteady flow equation and the peak of water level) was plotted against the hydrograph parameter and is shown in Figure 4. This graph does not show any proper correlation between these two parameters, however, all data points are confined to a narrow range of relative time lag varying between 0.02 and 0.10.

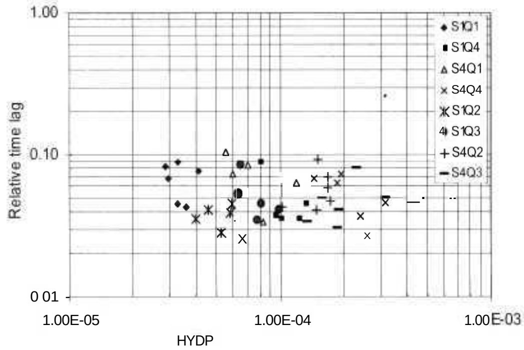


Figure 4: Relative time lag Vs hydrograph parameter

Figure 5 illustrates the time variation of change in shear velocities predicted by the unsteady and steady state formulas during the passage of a hydrograph.

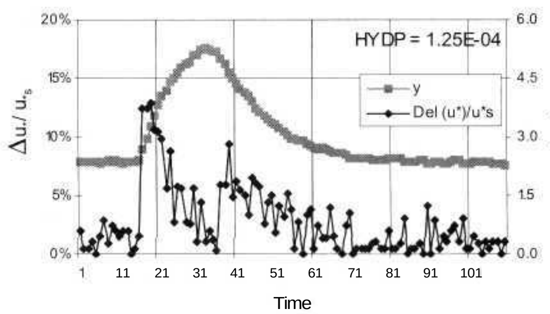


Figure 5: A typical plot of time variation of $\Delta U^*_{un}/U^*_s$ during the passage of a hydrograph.

It can be noted that $\Delta u_* / u_{*s}$ is larger in accelerating flow than that in decelerating flow and the difference is very small in the base flow. The largest $\Delta u_* / u_{*s}$ in the rising and falling branches of a hydrograph appeared to occur at locations where the rate of change of water level was high.

This can also be shown theoretically as follows (Kabir, [6]);

Using Eqns. (14) and (15);

$$\frac{u_{*un}}{u_{*s}} = \sqrt{1 + \frac{1}{CS}\frac{\partial y}{\partial t}} \qquad (17)$$

where u_{*un} and u_{*s} are the shear velocities given by unsteady and steady flow equations, respectively and $C = U + \sqrt{gy}$. The above equation can also be written as;

$$\frac{u_{*un}}{u_{*s}} = \sqrt{1 + \frac{1}{US}\left[\frac{Fr}{1+Fr}\right]\frac{dy}{dt}} \qquad (18)$$

where $Fr = U/\sqrt{gy}$. According to Eq.(18), during the rising limb of hydrograph, where $\partial y/\partial t > 0$, therefore $u_{*un} > u_{*s}$, whereas during the falling limb, where $\partial y/\partial t < 0$, and then $u_{*un} > u_{*s}$, as clearly illustrated in Figure 2. Defining the difference in shear velocities as,

$$\frac{\Delta u_*}{u_{*s}} = \left(\left|\frac{u_{*un} - u_{*s}}{u_{*s}}\right|\right) = \left|\sqrt{1 + \frac{1}{US}\left[\frac{Fr}{1+Fr}\right]\frac{\partial y}{\partial t}} - 1\right| \qquad (19)$$

Eq.(19) clearly indicates that the difference in shear velocities becomes maximum when $\partial y/\partial t$ reaches its peak values. Eq.(19) also gives the error for computing the shear velocity in unsteady flows, if the steady state formula (Eq.(15)) is used. This error is directly proportional to $\partial y/\partial t$ and inversely proportional to the stream-power *(US)*. This information will be very useful for sediment transport studies in unsteady flows.

Figure 6 shows the variation of maximum error for computing shear velocity using steady state formula, $(\Delta u_* / u_{*s})$ with *HYDP* in the rising and falling branches of the hydrographs used in this study. Although the data points are somewhat scattered, it can be noted that the maximum error is generally larger in the rising branch *(accelerating flow)* than that in the falling branch *(decelerating flow)* and it always remains at less than 3% in the base flow. The maximum $(\Delta u_* / u_{*s})$ increases with *HYDP* indicating that the error can be as high as nearly 30% and 20% in the rising and falling branches, respectively when HYDP is high.

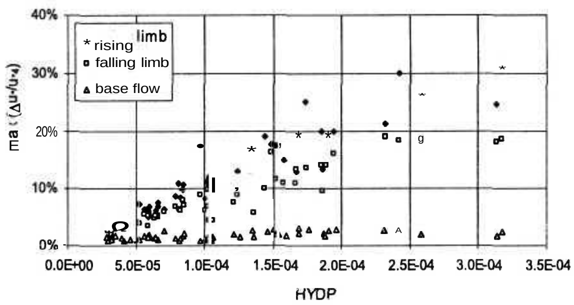


Figure 6: Variation of maximum ($\Delta u_* / u_{*s}$) with HYDP in the rising branch (accelerating flow) and falling branch (decelerating flow) of the ***hydrographs.***

4. Conclusions

Detailed studies were carried out to investigate the behaviour of bed shear stress (which is directly related to shear velocity) in unsteady flow using laboratory experiments performed in an open channel over a rough bed. The shear velocity in unsteady flow was computed using de Saint Venant equations and it was compared with the steady state formula. The following conclusions can be made from this study:

(i) For a given hydrograph, the shear velocity is usually larger in the rising branch than that in the falling branch.

(ii) Shear velocity always reaches its maximum value before the water level reaches its maximum value.

(iii) The difference in shear velocities predicted from steady and unsteady flow equations was quantified and the difference appeared to increase with unsteadiness of the flow (HYDP).

(iv) In the rising and falling branches of hydrographs, the difference in shear velocities is largest at locations where the rate of change of water level is high. The shear velocities are almost the same at water level peaks and in the base flow.

(v) A relationship between steady and unsteady shear velocities was derived in terms of hydraulic parameters of the problem. This equation can be used to quantify the error for computing shear velocity in unsteady flows, if steady state formula is used.

Acknowledgements

The authors wish to acknowledge the financial support given by the National Science Foundation, Sri Lanka, Grant Number CRG/ 98/P/02 for conducting the experimental runs. The second author wishes to gratefully acknowledge the scholarship received from the Science & Technology Personnel Development Project of the Ministry of Science & Technology, Sri Lanka for his postgraduate studies.

References

1. Cardoso A.H., W.H. Graf and G. Gust "Steady gradually accelerating flow in a smooth open channel", /. *Hydr. Res.,* Vol.29, No.4, pp.525 - 543, 1991.
2. Chang, HH: "Fluvial Processes in River Engineering". John Wiley and Sons, Inc, 1988.
3. Chien N, Z. Wan: "Mechanics of Sediment Transport", The American Society of Civil Engineers, ASCE Press, 1998.
4. Graf WH, T. Song: "Bed-shear stress in non-uniform and unsteady open channel flows", J. Hydr. Res., Vol.33, No.5, pp.699 - 704, 1995.
5. Henderson FM: "Flood waves in prismatic channels", /. *Hydr. Engrg.,* ASCE, Vol. 89(4), pp.39 - 67, 1996
6. Kabir MR: "Bed load transport due to unsteady flow", PhD Dissertation, Katholieke Universiteit Leuven, Belgium, 1993.
7. Kabir MR, H. Trofs: "Comparison of different methods to calculate bed shear stress", *Wat. Sci. Tech.,* Vol.25, No.8, pp.131-140, 1992.
8. Keulegan GH: "Laws of turbulent flows in open channels", /. *Res., Nat. Bureau of Standards,* Vol.21, No.6, pp.707-741, 1938.
9. Pathirana K.P.P., P.C. Ranasighe and U.R. Ratnayake: "Influence of Reference Bed Level on Computation of Bed Shear Stress in Open Channel Flow", *Engineer,* The Institution of Engineers, Sri Lanka, Vol.XXXIX, No.03, pp.19-24,2006.
10. Song T., W.H. Graf, U. Lemmin: "Uniform flow in open channels with movable gravel bed", /. *Hydr. Res.,* Vol.32, No.6, pp.861 - 876, 1995.
11. Tu H. and W.H. Graf: "Friction in unsteady open-channel flow over gravel beds", /. *Hydr. Res.,* Vol.31, No.1, pp.99 - 110, 1993.