

Optimum Design of Prestressed Concrete Beams

P. B. R. Dissanayake and S. Jothy Karma

Abstract: One of the motivations in optimizing structures is to reduce the overall weight, which often results in reduced material cost. Efficiency implies minimum cost or minimum weight while satisfying a variety of strength and stiffness requirements. In all engineering problems, designers try to find solutions giving good performance, which satisfy several requirements. Using optimization techniques, engineers can obtain the optimum results, within the imposed conditions. Structures designed in this way are safer, more reliable and less expensive than the traditional designs.

This paper describes how MSEXcel is utilized to organize, manage and direct for solving and optimizing a pre-stressed concrete beam section. To evaluate the stress and deflection, SAP2000 structural analysis software is used. This numerical design optimization provides the designer with a computational tool that finds the best design, based on predefined performance requirements. MSEXcel optimizer automatically makes changes to problem parameters that are allowed to vary, referred to as design variables and performs a new analysis (linear or non-linear) to evaluate the influence of the changes, repeating the process until the design that best satisfies the performance requirement is found. The optimized results are then compared with manual design results.

Keywords: Optimization, Numerical, Model

1. Introduction

Modern engineering consists of a number of well-established activities, including analysis, design, fabrication, sales, research and the development of systems. The design of systems is a major field that has been developed and used for centuries, and the existence of fine buildings, bridges, highways, automobiles, airplanes, space vehicles and other complex systems are excellent testimonials. However, evolution of these systems has been slow. The entire process is both time consuming and costly, requiring substantial human involvements.

Several systems can usually accomplish the same task, and some are better than others. For example, the purpose of a bridge is to provide continuity in traffic from one side to the other. Several types of bridges can serve this purpose. However, to analyze the possibilities can be a time-consuming and costly affair. Usually one type is selected and designed in detail.

The design of complex systems requires a large amount of calculation and data processing. During the last three decades, a revolution in computer technology and numerical computations has taken place. Today's computers can perform complex calculations and process large amounts of data efficiently. The engineering design process benefits greatly

from this revolution. Better systems can now be designed by analyzing various options in a short time. This is highly desirable because better-designed systems cost less, have more capability, and are easy to maintain and operate.

Optimal design of structures is always a goal of engineers, whether or not mathematically based approaches are used to drive optimization procedures. The current practice in most building designs are optimized by trial and error combined with the experience of the designer. It is a time consuming process, but using optimisation algorithms with computer programmes, time consumption can be minimized. In general, optimum design problems seek to minimize a function (usually cost) using a set of design variables subjected to constraints.

2. Design Problems

Methodologies and algorithms are used to create lighter, stronger, and safer structures. These algorithms form the basis for *structural optimization*. The design of a pre-stressed beam requires a systematic sequence of trial- and-error steps.

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1. An initial trial design is assumed based on the given parameters of the pre-stressed beam (e.g pre-stressed beam topology, loading conditions, and material properties).
2. The response of the pre-stressed beam is determined via structural analysis (Using SAP2000).
3. The pre-stressed beam response is evaluated with respect to governing design specifications.
4. A new design is selected to eliminate any violations of the specifications and/or improve the economy of the pre-stressed beam.

The trial-and-error process is continued until an acceptable design is attained. The efficiency of the trial and error procedure is greatly dependent on the experience and ability of the designer to select a "good" initial design and perturb the design in a "better" direction towards an optimal design. As a result, techniques of optimization have developed to automate and mathematically orchestrate the trial and error procedure.

Optimization techniques can be classified as gradient-based (requiring continuous variable representation) and direct search techniques (often using discrete variable representation). The Microsoft Excel Solver tool uses the Generalized Reduced Gradient (GRG2) nonlinear optimization code developed by Leon Lasdon, University of Texas at Austin, and Allan Waren, Cleveland State University.

The example used here is to find the optimum cross sectional dimensions of a pre-stressed concrete beam and its pre-stressing force. Design variables are $b_1, b_2, b_3, t_1, t_2, t_3, P$, and e as shown

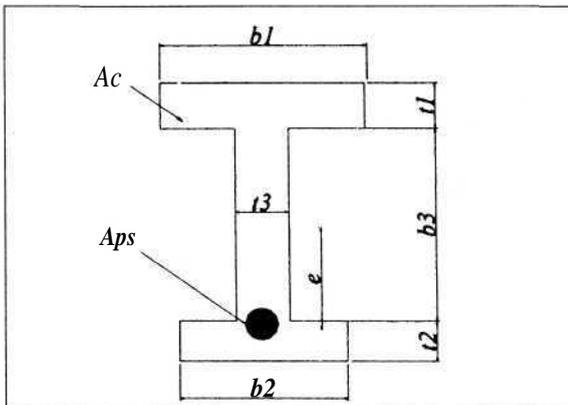


Figure 1- Cross Section of the Pre-Stressed Beam

in Figure 1. The aim of this paper is to find these design variables with respect to several spans and different load cases. The constraints set in this problem are allowable tensile and compressive stresses. Those were considered at transfer and service conditions; allowable deflection and shear capacity were also considered.

3. Formulation Of The Design Problem

General optimisation problem can be summarized as follows;

$$\begin{aligned} \text{Find} & \quad X \\ \text{To minimize} & \quad Z(X) \\ \text{Subject to} & \quad g(X) \\ & \quad X^l \leq X \leq X^u \end{aligned}$$

Where X is the vector of design variables, $g(X)$ are design constraints, X^l is lower bound of the design variables and X^u is upper bound of the design variables.

The design problem, formulated for a pre-stressed concrete beam is as below;

Find $b_1, b_2, b_3, t_1, t_2, t_3, P, c$, which

$$\text{Minimize } ACL + A_{ps} C_s L = Z(b_1, b_2, b_3, t_1, t_2, t_3, P, c) \quad (1)$$

$$\text{Subject to } \sigma^t - \sigma_{at}^c < 0 \quad (2)$$

$$\sigma^c - \sigma_{ac}^c < 0 \quad (3)$$

$$V - V_c \leq 0 \quad (4)$$

$$X \leq X_0^u \quad (5)$$

$$X \leq -X_0^l \quad (6)$$

Where,

A_c - Cross section area of the section
 $= b_1 t_1 + b_2 t_2 + b_3 t_3$

C_c - Cost of concrete per unit volume

A_p - Cross section area of the steel
 $= P / \sigma_{steel, allowable}$

C_s - Cost of steel per unit volume

L - length of the pre-stressed beam

σ^t, σ^c - Maximum Tensile, Compressive stresses, respectively in concrete

σ_v, σ - Allowable Tensile & Compressive stresses, in concrete

V_c - Ultimate shear stress

V - Applied shear stress

The beam is simply supported at the ends and is subjected to a uniformly distributed load (udl) and a point load at the center.

Table :1 Input data of the problem

Bridge Data	Value	unit
Total Length	30	m
Width of the bridge	7200	mm
One lane width	3600	mm
No.of Beams per lane	2	
Height of insitu concrete	200	mm
OBJECTIVE FUNCTION Z (MINIMIZE)	144302	Rs.

Design Variables

b_1	454.91	mm
t_1	225.00	mm
b_2	327.00	mm
t_2	225.00	mm
b_3	1830.00	mm
t_3	152.00	mm
P	3522.26	kN
e	836.70	mm

Material Data

Strength of pre cast concrete	50	N/mm^2
Strength of insitu concrete	35	N/mm^2
Yield strength of tendon	1770	N/mm^2
Nominal strength of tendon	1239	N/mm^2
Density of concrete	24	kN/m^3
prestressed loss ratio , n	0.85	

Loading

Self weight at Transfer condition	10.89	kN/m
Self weight at Service condition	19.53	kN/m

Imposed loads

HA loading udl per lane	30	kN/m
HA loading point load per lane	120	kN
HB Load Case	25	units

The above inputs were fed to the *MS-Excel/Solver* programme. After getting the optimum design variables and pre-stressed force, Finite Element Model of the beam was developed by using *SAP2000*, to check its performance under different conditions.

4. Analysis Using SAP2000

In the case study a 28.2 m span pre-stressed beam was considered. Following is a description of inputs in to the SAP2000 FE model.

Number of Nodes	= 200
Number of 8 Node Elements	= 92
Number of materials (Concrete, steel)	= 2
Number of load cases	= 3

The Following results were obtained by using SAP2000

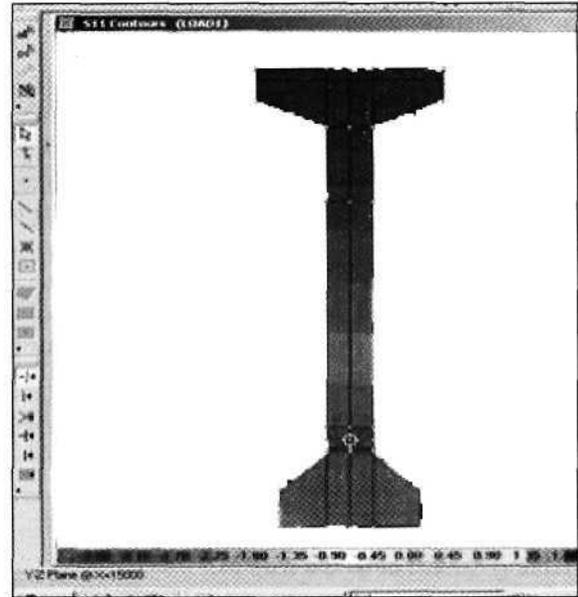


Figure 1- Stress along the Cross Section of the Pre-Stressed Beam

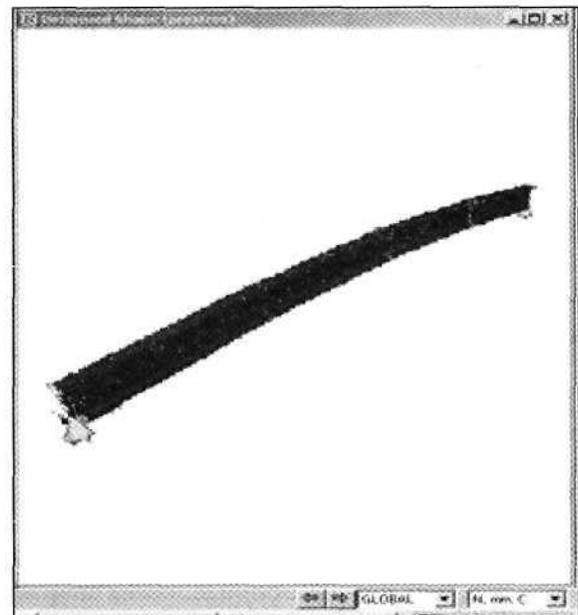


Figure 3- Deformed shape due to Pre-stressed force

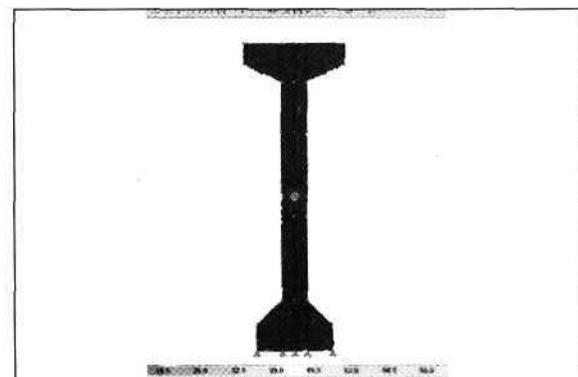


Figure 4- Stress along the Cross Section of the Pre-Stressed Beam At the Support

5. Results

Using *Solver* tool in MS-Excel optimum variables were obtained. The optimum geometry results and pre-stress force was modeled in *SAP 2000*. In all load cases, stresses of every element and deflection of every node were checked in *SAP 2000* model.

Using the MS-Excel programme, considering *HA* loading case the following optimum dimensions, pre-stressing forces and eccentricities were obtained, with $C = \text{Rs. } 9000.00$, $C_s = \text{Rs. } 528900.00$, $\sigma_{st} = 2.13 \text{ N/mm}^2$, $\sigma_{nc} = 20 \text{ N/mm}^2$, $V_c = 5.3 \text{ N/mm}^2$

Table 2: Results obtained with MS-Excel

SPAN(m)		10	20	28.2
IMPOSED LOAD	udl	15	15	15
	Point	60	60	60
b_1 (mm)		280	357	379
b_2 (mm)		300	406	327
t_1 (mm)		122	235	225
t_2 (mm)		150	289	225
t_3 (mm)		152	152	152
b_3 (mm)		482	608	1830
P(MN)		1.28	2.7	2.85
e(mm)		235	442	878
Z (Rs)		17159	69492	138678

The results obtained with MS-Excel is given in Table 2.

6. Conclusion

This paper demonstrates through a case study how engineers can use optimization techniques for their designs using MS-Excel (Solver).

Acknowledgement

The authors would like to give their sincere acknowledgement to the National Science Foundation of Sri Lanka for the financial support given through the research grant numbers RG/2003/E/001 & NSF/SCH/2005/03

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