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A Model for the Prediction of Strength Enhancement in Concrete Cores under Passive Lateral Confinement

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Abstract: A behaviour-oriented model is proposed for the confinement effectiveness and strength enhancement of concrete under axial compression, passively confined by lateral reinforcement. The model is developed on the basis of three dimensional concrete constitutive law based FEM simulation results and idealized experimental results on laterally confined concrete cores. Confinement efficiency of lateral reinforcement layouts are quantified through a mechanically defined confinement effectiveness index, which is the ratio of induced spatial average confining stress to the maximum potential confinement capacity of lateral reinforcement arrangement. This index is represented by lateral reinforcement detailing as well as material parameters in the model. Influence from uniformity of confining reinforcement layouts on confinement effectiveness was identified through FEM simulation results, which is used to formulate the basic form of the model. Concept of lowest confined section is adopted to formulate the strength enhancement due to confinement, based on average confining stress and double effect of uniformity of lateral confinement layout.

Keywords: model, confinement, concrete compressive strength, strength gain

1. Introduction

Concrete under active lateral confinement is known to produce enhancement in both strength and ductility under axial compression. Experimental as well as analytical research conducted during the past couple of decades indicated that passive lateral confinement too could significantly contribute in the enhancement of strength and ductility in concrete. Since confinement of concrete by laterally placed reinforcement inherently can only provide passive confinement, i.e. confinement is generated only when concrete laterally expands against the reinforcement; this finding becomes important in reinforced concrete mechanics.

For a brittle material such as concrete, mainly utilized to resist compressive loads in structural applications, enhancement of strength in compression coupled with increased capacity for deformation or ductility is significant. Especially for structures in seismic zones, this enhanced capacity would provide safety and serviceability under seismic overloading. Due to these beneficial effects on the performance of concrete resulting in better safety, economy and applicability, lateral confinement has received much attention in research. Lateral confinement is generally provided to a concrete core by steel reinforcement in the form of casings or discretely placed ties and hoops. Several experimental investigations have been conducted on scaled down as well as nearly full size discretely confined concrete columns under axial compressive loading [1, 2, 7, 11, 12]. Through these studies, several empirical or semi analytical models on strength enhancement of confined concrete have been proposed. These models are seen to produce good predictions within their application domain, generally decided by the range of parameters considered in the respective experimental investigation. However, since a wide range of detailing and material parameter variations are possible in a laterally confined concrete column, applicability of these models outside their development domains are not very convincing. Reason for this could be inadequate portrayal of significant material as well as detailing parameters in the proposed relations. Proper identification of the sensitivity of governing parameters requires a rational method of addressing the confinement mechanism.

As a development on the microscopic understanding of complex non-linearity of concrete, a 3-Dimensional non-linear constitutive law has been proposed [3, 4, 5, 6]. The constitutive equation for concrete known as

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'Elasto-plastic & Fracture model' is based on mathematically quantified four experimental facts namely; Fracture in hydrostatic stress state, Fracture in shear, Plasticity in shear and Plasticity in volume [9]. Since this constitutive model was developed with particular emphasis on confinement phenomenon, it can be applied to simulate structural member behaviour through the Finite Element Method (FEM) approach. This FEM based microscopic method was verified on member level applicable range [9] based on results of an idealized experimental investigation on square, purely lateral confined concrete cores under axial compression [8].

For the FEM modelling of confining steel, two idealizations were used. First idealization based on the assumption of steel reinforcement only being able to provide axial stiffness, was modelled with a three-dimensional isoparametric element with only translational degrees of freedom at nodes. This idealization was applicable for reinforcement in circular confined cores. The second idealization, which additionally accounted for flexural and shear stiffness of reinforcement was modelled using 'Timoshenko beam element' with both translational and rotational fields interpolated along the element to the nodes. With this model, lateral reinforcement in square sections, capable of providing confinement by flexure and shear in addition to axial stiffness, was idealized. By adopting this idealization the effect of differing cross sections of confining steel could be accounted.

In this experimental investigation, significant parameters based on amount and spacing of discrete lateral reinforcement were addressed in a wide range. Due to the versatility of the FEM analytical approach, it can be applied to investigate the sensitivity of detailing and material parameters on the strength enhancement by confinement.

It is the aim of this paper to combine micro-mechanical FEM analysis results on trends and sensitivity of governing parameters with experimental results, in developing a "behaviour oriented model", for predicting the confinement effectiveness and strength capacity of laterally confined concrete cores. The most significant concept identified through analytical FEM investigations on confinement is the influence of non-uniformity in the confining layouts. Due to this non-uniformity, weaker sections are created which result in the reduction of confinement effectiveness in terms of both developed average confining stress and strength enhancement. In the derivation of the model on the strength enhancement, this effect of non-uniformity is explicitly incorporated at two well-identified levels. The first effect is applied in the development of the averaged confining stress over the concrete domain based on the stresses generated in lateral reinforcement. Second influence governs the strength enhancement based on the induced average confining stress.

2. Average Confining Stress by Lateral Reinforcement

Confinement effectiveness index is defined as the ratio of spatial average confining stress developed at the peak strength of the core (σ_v) to the potential confinement capacity ($^1/_2 pf_y$) [8]. Potential confinement capacity is attained only when all lateral steel reaches yield condition. Based on FEM analysis results [9] and idealized results of experiments [8], following parameters were identified as the most influential on the confinement effectiveness.

1. The amount of lateral reinforcement expressed in terms of volume ratio of lateral steel to that of the confined concrete core ($p = V_s/V_c$). Here, the confined core is defined as the concrete area bounded by the centrelines of the peripheral lateral tie.
2. The spacing of lateral reinforcement given in non-dimensional parameter of spacing ratio (s/d). This is the ratio between center-to-center spacing of lateral ties and the least lateral dimension of the core.
3. The contribution of the flexural stiffness of lateral reinforcement arms, expressed as the ratio of effective diameter of the tie bar cross section to the span of the tie arm (φ/L).

Apart from the main parameters above, effect of unconfined concrete strength on the confinement effectiveness identified through micro-mechanical study results is incorporated in the development of the model. Furthermore, the steel stress-strain characteristics such as yield strength, and elastic modulus will have significant bearing on the confinement effectiveness. However, for the range of hot rolled steel properties (f_y = 300-350 MPa) used for the development of the model through experimental results, this effect can be neglected since steel yield strains are within a close range.

The basic mathematical form of model for the confinement effectiveness index is proposed as depicted in eq. 1. This equation does not attain unrealistic values at extreme conditions such as zero spacing, as observed in some previous design equations.

$$\alpha = \frac{1}{1 + K^{*} + K_0 (s/d)^{y}} \quad \text{.......} \quad (1)$$

In this form, the factor K^* termed the "Limit Uniformity Factor" represents the condition when spacing becomes zero. In other words, K* indicates the limit case in which the uniformity along the axis of the concrete core is fully attained. This factor is intended to represent the material strength as well as uniformity of confinement stress distribution at lateral reinforcement level. Increase in this factor indicates increase in non-uniformity due to material strength as well as the shape and flexural stiffness of the confining agent in the lateral direction.

The parameter K_0 termed the "Uniformity Factor" accounts for the uniformity of confining stresses in the lateral direction when the spacing of lateral reinforcement becomes larger than zero. Increase in this factor indicates an increase in the lateral non-uniformity based on shape and flexural stiffness of the confining agent. Under discretely placed lateral reinforcement condition, uniformity in the axial direction is portrayed by the lateral reinforcement spacing ratio.

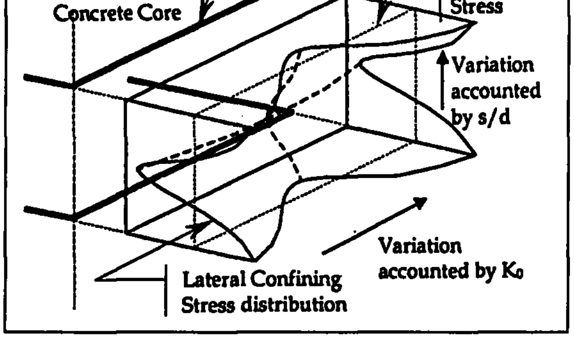

Figure 1. Confining stress distribution by discretely spaced ties

Therefore, uniformity factor multiplied by the spacing ratio is adopted for representing the overall lateral stress distribution. This discussion is graphically represented in Fig. 1, through a conceptual distribution of lateral confining stresses. This representation is based on the trends observed in the idealized FEM analysis results.

With emphasis on confinement uniformity, the tendencies of the proposed equation in limit conditions as well as overall range are depicted in Fig. 2. Full sectional stress uniformity offered by a circular tie accounted as a limit condition is shown in the figure.

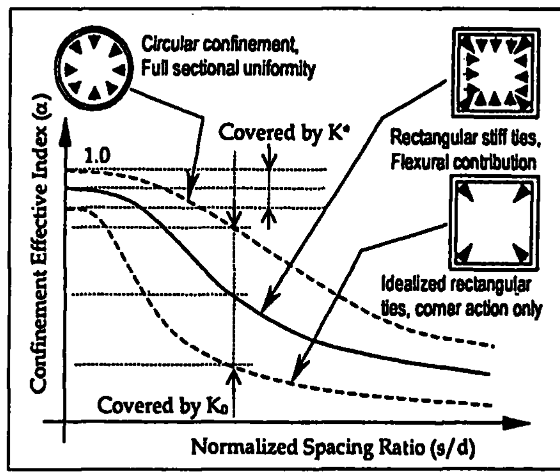

Figure 2. Range of the model for confinement effectiveness (illustrated)

2.1 Derivation of Limit uniformity factor

The limit uniformity factor accounts for the condition when the spacing becomes zero, which can be viewed from another perspective as the condition when complete axial uniformity is maintained.

As the limiting condition of zero spacing for discretely spaced lateral reinforcement, experiments based on casing confined concrete with longitudinal stiffness of steel is not appropriate since it invariably contributes in the axial load carrying mechanism. Though the experimental method cannot be applied effectively in this matter, analytical method based on FEM can be used with intentionally removed load-carrying properties in the axial direction.

It is clear that in the case of a square casing, the flexural stiffness of the sides between corners will contribute to confinement. However, when the thickness of the sides becomes very small compared with the lateral breadth of the casing, confinement can only be transferred effectively at the corners. This situation can be assumed closer to FEM modelling of steel as truss elements with negligible contribution of flexural stiffness in confining action. Therefore, the analytical results with steel as truss members were utilized for the development of the basic form for limit uniformity factor.

For normal yield strength range of steel used for lateral reinforcement in this study (300 - 350 MPa), the influencing parameters on limit uniformity factor were identified as the potential confinement capacity of lateral reinforcement ($^1/_2 pf_y$) and the unconfined compressive strength of concrete (f_{co}), by FEM analysis. Based on analytical results eq. 2 is proposed for the limit uniformity factor of square casing confined concrete without the contribution of confinement by flexural stiffness of ties.

$$[K^*]_{sq.casing} = 0.7\left[\frac{pf_y}{f_{co}}\right]^2 \qquad (2)$$

Where, p is the volumetric lateral reinforcement ratio and f_y is the yield strength of steel. The proposed relation as compared with the analytical results at three levels of unconfined concrete compressive strengths are shown in Fig. 3, against lateral reinforcement ratio.

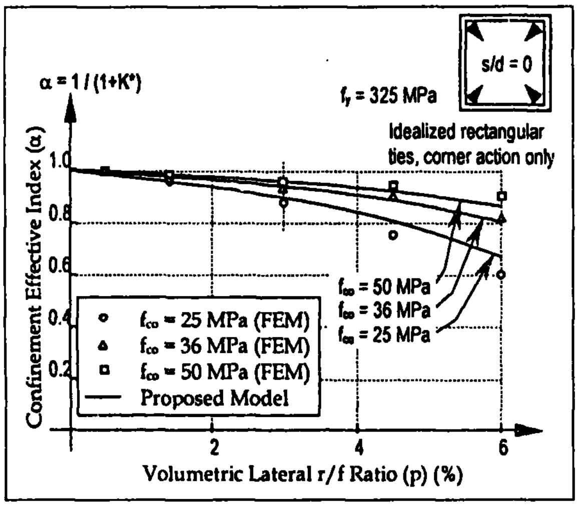

Figure 3. Limit uniformity factor (material)

To extend the obtained idealized condition to general situation of a square casing with flexural stiffness of the sides, FEM results using Timoshenko's beam idealization of steel is used. Instead of casing, results of discretely placed ties with very low spacing ratio are utilized here.

The contribution of flexural stiffness of ties is accounted through a relation termed "flexure/shape factor" (F_r) as given by eq. 3 based on experimentally observed results. Increase in this factor implies a higher uniformity of confinement on the core. As the name implies, this factor accounts for the shape of the lateral reinforcement by considering variations in linear span of the reinforcement arms. Conceptually it can account for the higher confinement uniformity afforded by hexagonal, octagonal or circular tie configurations.

$$F_r = 1 + 350\left[\frac{\varphi}{L}\right]^2 \qquad (3)$$

Where, φ is the effective diameter of the tie cross section, explained later and L is the linear span of a tie arm. It should be noted that for the case of a circular section where the linear span of a tie arm becomes conceptually zero, a limiting value of φ/L ratio (= 1/5) is assumed (i.e. F_r = 15). This presents a rational way to extend the above factor to account for circular confinement agents. Derivation of the flexure/shape factor and assumptions to account for shape will be discussed later.

Proposed complete relation for limit uniformity factor K^* is given by eq. 4.

$$K^* = \frac{0.7\left[\frac{pf_y}{f_{co}}\right]^2}{F_r} \qquad (4)$$

FEM analytical values compared with the relation are given in Fig. 4.

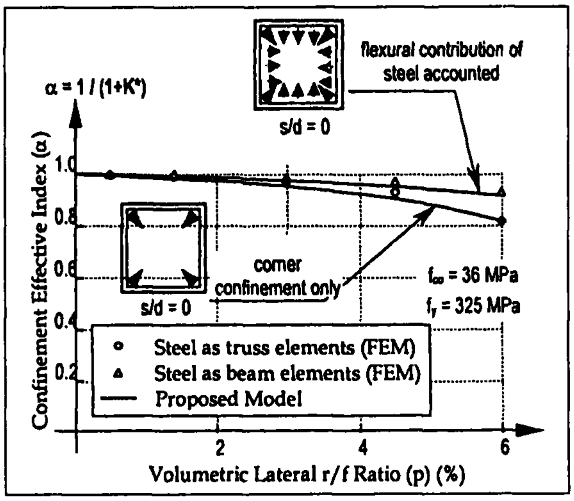

Figure 4. Limit uniformity factor (detailing)

2.2 Derivation of Uniformity factor, K0

Uniformity factor depicts the lateral uniformity of confining stresses when the reinforcement is discretely placed. When this factor is coupled with spacing ratio, which indicates the axial confinement stress uniformity, the resultant gives an indicator of the uniformity of lateral confining stress distribution as presented in Fig. 1. The uniformity factor is designed to increase with the evolution of non-uniformity so that the product of this factor with spacing ratio gives a total indicator of lateral non-uniformity.

Highest lateral non-uniformity across a confined concrete section is observed when the idealized condition of a square section only confined at the corners is considered. This idealized condition behaviour can be appraised by the FEM analysis results using steel as truss elements in the simulations. In this condition, no beneficial contribution to confinement by flexural stiffness of the ties is considered. Therefore to identify the upper limit of the uniformity factor indicating highest non-uniformity, this ideal condition is appropriate. Most uniform lateral confinement stress distribution across a section can be assumed for a circular section, which would govern the other limiting condition of the uniformity factor. Under this condition the uniformity factor will attain its lowest value.

With consideration of both experimental and analytical variation of confinement effectiveness index, uniformity factor was formulated as a direct function of flexure/shape factor F_r as given in eq. 5.

$$K_0 = \frac{7}{F_r} \quad \text{..........} \quad (5)$$

In conjunction, exponent γ in eq. 1 was decided to be a constant value of 3 to represent the effect of spacing ratio. With the incorporation of above conditions, the behaviour of the model for the confinement effectiveness index is depicted under varying spacing ratio in Fig. 5. In this figure, FEM analytical results of two limiting conditions of idealized corner action confined square section and a circular section are compared.

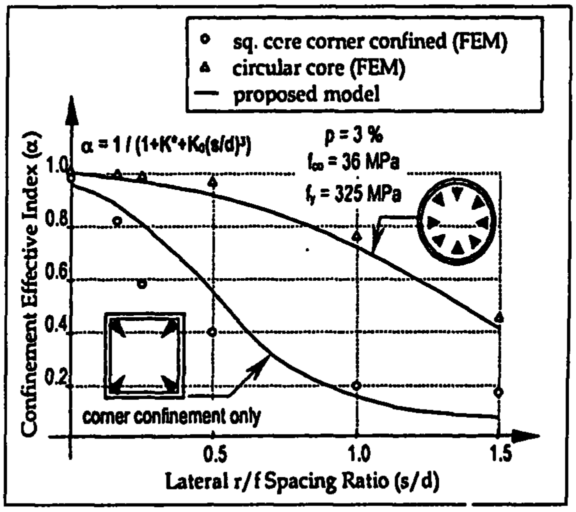

Figure 5. Physical range of confinement effectiveness index

2.3 Derivation of Flexure/shape Factor

Flexure/shape factor as utilized in previous sections accounts for the contribution of confinement action parted by flexure and shape of tie arms. The basic parameter in this factor is the square of the ratio of effective tie diameter to the linear span of the arm. The effective tie diameter is considered since tie bar cross section can be other than circular (e.g. rectangular).

Basic parameter is based on the ratio of flexural stiffness to axial stiffness of a tie member under elastic conditions as given by eq. 6. Here, L is the span of tie arm bounded by corners with its cross sectional area and moment of inertia given by A and I, respectively.

$$\frac{\textit{Felxural stiffness}}{\textit{Axial stiffness}} \propto \frac{I/A}{L^2} \quad \text{..........} \quad (6)$$

For a circular tie bar cross section, this ratio becomes proportional to the square of the ratio of bar diameter to the span of the tie arm, as given by eq. 7.

$$\frac{\textit{flexural stiffness}}{\textit{axial stiffness}} \propto \frac{1}{16}\left[\frac{\varphi}{L}\right]^2 \quad \text{..........} \quad (7)$$

Considering the circular tie bar cross section to be the basis for shape, the square of the ratio of tie diameter to arm span was selected as the governing parameter. For other cross section ties the effective diameter (φ_{eff}) is given by eq. 8.

$$(\varphi)_{eff} = 4\sqrt{\frac{I}{A}} \quad \text{..........} \quad (8)$$

It is seen that (φ/L) factor can have a value close to zero when the span becomes very large compared with the effective diameter of the bar. On the other hand, the upper limit of this factor is selected to represent the condition of full sectional uniformity provided by a circular tie. This upper limit is taken as $\varphi/L = 1/5$ which results in $F_r = 15$. This provides a convenient way to transform the flexural stiffness term in the case of square ties to account for the condition governed by hoop stress in circular spiral or hoop confined columns.

Due to symmetry in the case of square sections confined with simple square ties, all tie arms have the same flexural contribution. Therefore, the ratio can be based on one tie arm. But when complex tie arrangements and/or different diameter ties are used at the same level, a weighted average of the above ratio is proposed to account for the contribution of flexural stiffness and shape of the lateral reinforcement system. The weighting is based on the length of each arm and is expressed in eq. 9.

$$\left[\left(\frac{\phi}{L}\right)^2\right]_{eq} = \left[\frac{\sum_{i=1}^{n}(\phi_i / L_i)^2 L_i}{\sum_{i=1}^{n} L_i}\right] \quad (9)$$

Based on the weighted equivalent ratio discussed above, the flexure/shape factor, which signifies the contribution of tie flexural stiffness, is defined as given in eq. 10.

$$F_r = \left[1 + 350\left[\left(\frac{\varphi}{L}\right)^2\right]_{eq}\right] \quad (10)$$

This relation can be used to evaluate the overall contribution when different types of ties are applied at the same level (e.g. square and octagonal ties).

Comparison of experimentally observed results of confinement effectiveness index for varying spacing ratio with predictions by the proposed model is shown in Fig. 6. Dashed lines in the figure indicate paths of variation of confinement effectiveness under the same flexural stiffness of the tie, as computed by the model. It is seen that the experimental results corresponding to each bar diameter closely match the corresponding point on the trend curve predicted by the model.

When the confinement effectiveness index is known, spatial average confining stress at the peak strength of the core concrete can be computed based on the amount of lateral reinforcement presented as a volume fraction of the core under confinement and the yield strength of steel as given by eq. 11.

$$\sigma_v = \alpha \frac{1}{2} p f_y \quad (11)$$

The predicted confinement effectiveness indices by the model and experimentally measured average confining stresses at peak strengths of respective cores, for the specimens from the comprehensive experimental programme [10], are compiled in Tab. 1.

Table 1 - Confinement related model predictions (at peak strengths of idealized concrete cores [10])

Designation of cores and comments	Lat. r/f ratio (p) (%)	Spacing ratio (s/d)	Confinement effectiveness Index (φ)		Average confining stress (σ_v) (MPa)	
			Exp.	Model	Exp.	Model
C16-075	5.70	0.41	0.80	0.84	7.64	7.99
D19-104	5.92	0.58	0.78	0.75	7.20	6.97
O19x2-232	5.37	1.20	0.20	0.29	1.72	2.44
A09-042	3.22	0.22	0.94	0.93	5.06	5.00
H13-094	3.05	0.51	0.73	0.73	3.67	3.67
I16-150	2.91	0.83	0.46	0.48	2.11	2.18
J19-225	2.77	1.26	0.24	0.26	1.07	1.13
M09-090	1.50	0.48	0.71	0.70	1.80	1.76
N13-192	1.49	1.04	0.26	0.26	0.64	0.62
P09-043 small core	4.26	0.31	0.90	0.90	6.38	6.34
S25-119 large core	4.35	0.32	0.85	0.89	5.63	5.89
T13-065	4.40	0.35	0.91	0.87	6.56	6.27
V16_075 Low f_u	5.81	0.41	0.79	0.82	7.18	7.48

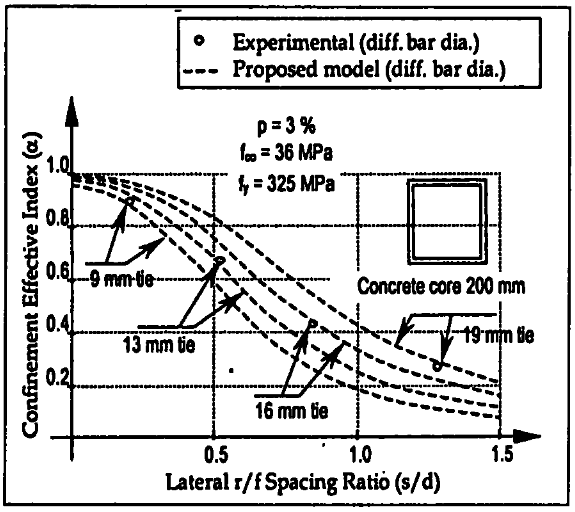

Figure 6. Model prediction of experimentally evaluated confinement effectiveness index for different tie stuffinesses

3. Strength Enhancement

The spatial average confining stress depicts the average of varying confining stresses developed by discretely applied agents as illustrated in Fig. 1. It is rational to assume that the weakest confined section governs the peak strength of concrete. This weakest section can be considered at the midway position of two discrete ties. Conceptually, the average confining stress condition at this section, which is termed the minimum sectional average confining stress, will be the governing condition for the peak strength of the confined section. Though the location of this weakest section can be identified, the stress condition prevailing at the level cannot be experimentally measured.

Whereas it is impossible to measure the stress condition at the weakest or the critical section experimentally, it is possible to compute a representative value for this conceptual quantity using micro-mechanical FEM analysis. Therefore, FEM method can be used as a parametric study to identify the relation between the spatial average confining stress, minimum sectional average confining stress and strength gain. It was experimentally observed that even though two columns with different tie spacings develop the same average confining stress, the strength gains were different. The spacing has primary effect on the average confining stress as discussed through previous section. Then the above observation directly implies that the spacing has double effects on the peak strength of confined concrete. The relationship between the minimum sectional average confining stress or the critical confinement level and strength gain can be considered analogous to tri-axial confinement and associated strength gain which is only dependent on the uniform lateral confining stress level which also is the minimum sectional average confining stress. If this analogy were accepted, the relationship between the minimum sectional confining stress and the strength gain would be independent of the detailing or material parameters. Therefore, the transformation of the spatial average confining stress to the conceptual minimum sectional confining stress should account for the second effect of spacing.

A more uniform lateral stress distribution at a sectional level will be exhibited by a column confined with ties having higher flexural stiffness than in a column with ties of lower flexural stiffness. When two such columns, one with higher sectional stress uniformity than the other, are considered to have the same average confining stress as well as the same tie spacing, it is rational to assume that the former would develop a higher minimum sectional average confining stress resulting in a higher strength gain.

It is apparent from the above discussions that at least two influencing detailing parameters might govern the transformation of spatial average confining stress to conceptual minimum sectional confining stress. In order to identify any possible effects of these two parameters on the strength gain under confinement, an idealized study was conducted using micro-mechanical models through FEM simulations.

3.1 Effect of Tie Spacing

Two square concrete cores confined by square lateral ties were adopted for the study. Different tie spacings were applied to the two specimens. For comparison, these two specimens should develop the same average confining stress level at the peak axial strength of the concrete core. A trial and error process is invariably required for fulfilling the above conditions. In order to construct this process, volumetric reinforcement ratio was varied in the analytical simulation procedure. Results of the matching two cases are reported in Tab. 2.

Table 2. Strength gain under same average confining stress with varying tie spacing (FEM simulation)

Type	Average confinings tress (σ_l) (MPa)	Spacing ratio (s/d)	Dia. of tie (mm)	Lat. r/f ratio (p) (%)	Minimum sectional confining stress (σ_m) (MPa)	Stren. gain of core (Δf_c) (MPa)
Lower spacing	4.10	0.22	8.90	2.78	3.53	9.26
Higher spacing	4.10	0.51	17.5	3.05	3.15	8.17

The column with lower lateral reinforcement spacing had both low reinforcement ratio as well as lower lateral tie diameter, when the simulations matched the developed average confining stress levels. Higher spaced column certainly had a more uniform lateral stress distribution at tie level since a larger diameter bar with higher flexural rigidity was present. Nevertheless, the results indicate, both lower values for conceptual minimum sectional confining stress and lower strength gain for the higher spaced case. This observation clearly justify the assumption that higher lateral reinforcement spacing resulted in lower strength gain than lower spaced condition, even when the same average confining stress is developed.

3.2 Effect of Sectional Uniformity

The study of the effect of sectional uniformity requires the spacing and the average confining stress produced at the peak strength of the core to be the same. Differentiating parameter would be the lateral stress uniformity based on the shape or flexural stiffness of the tie arms. For this purpose one specimen idealized as corner confined with truss elements and the other with the confinement effect of contact action accounted by using Timoshenko beam elements were selected for FEM analysis.

Same spacing was used in both cases. The volumetric reinforcement ratio was used as the varying quantity for adjustment to match the developed average confining stresses. Detailing and results for the two cases by FEM simulations are compiled in Tab. 3.

It is clearly seen from the tabulation that, for the generation of the same average confining stress, a higher amount of reinforcement is needed by the corner-confined case. But, even under this condition, both strength gain and the representative minimum sectional confining stress are lower as compared to the normally confined case where flexural stiffness contribution of lateral reinforcement is accounted. Therefore, it can be concluded that, more uniform lateral stress distribution at tie level increases the minimum sectional average confining stress, through which the strength gain is increased.

Table 3. Strength gain under same average confining stress with varying sectional stress uniformity (FEM simulation)

Type	Average confining stress (σ_l) (MPa)	Spacing ratio (s/d)	Vol. r/f ratio (p) (%)	Minimum sectional confining stress (σ_m) (MPa)	Strength gain of core (Δf_c) (MPa)
Truss mechanism	3.20	0.25	3.80	2.36	6.07
Beam mechanism	3.20	0.25	2.32	2.82	7.78

3.3 Strength Gain by Confinement

Conceptual minimum sectional confining stress midway between adjacent ties was introduced to define a critical stress level which is directly associated with the strength gain due to confinement. The relationship between this critical stress level and the strength gain should not be affected by the material or detailing parameters. Furthermore, this relationship should be similar to the relationship between tri-axial lateral confinement and associated strength gain.

To achieve the above objectives, the transformation between average confining stress and the conceptually defined minimum sectional confining stress should account for the second effect of lateral reinforcement spacing as well as the uniformity of lateral confinement stresses at a tie level. This directly implies that the transformation is influenced by the lateral as well as axial confinement stress distributions. Similar parameters used in the derivation of the model for confinement effectiveness index in section 2., which are uniformity factor (K_0) and

Table 4. Strength related model predictions (at peak strengths of idealized concrete cores [10])

Desig nation and comments	Lat. r/f. ratio (p) (%)	Spac. ratio (s/d)	uncon. conc. stren. (f_{co}) (MPa)	Strength gain of core (Δf_c)(MPa)		Peak strength of core (f_{cc})(MPa)	
				Exp.	model	Exp.	model
C16-075	5.70	0.41	36.9	22.6	19.5	59.5	56.3
D19-104	5.92	0.58	35.6	17.9	18.1	53.5	53.7
O19x2-232	5.37	1.20	35.2	3.8	7.1	39.0	42.3
A09-042	3.22	0.22	35.6	10.8	12.0	46.3	47.5
H13-094	3.05	0.51	35.6	8.4	9.4	43.9	45.0
I16-150	2.91	0.83	35.6	7.0	6.4	42.6	41.9
J19-225	2.77	1.26	35.6	4.3	4.0	39.9	39.6
M09-090	1.50	0.48	35.2	5.1	4.6	40.3	39.8
N13-192	1.49	1.04	35.2	2.5	2.1	37.7	37.3
P09-043 small core	4.26	0.31	38.0	15.0	15.2	53.0	53.2
S25-119 large core	4.35	0.32	37.3	14.0	14.4	51.3	51.7
T13-065	4.40	0.35	36.7	15.1	15.2	51.8	51.9
V16_075 Low f_{co}	5.81	0.41	27.5	18.9	18.6	46.4	46.1

normalized tie spacing ratio (*s/d*), are adopted for the transformation with a minor modification.

Since the conceptual minimum sectional confining stress cannot be rationalized through experimental verifications, eq. 12 is a conceptual stepping-stone to attain the ultimate goal of the strength gain given by eq.13.

$$\sigma_{m(conceptual)} = \frac{\sigma_v}{1+0.6\,K_0\sqrt{(s'/d)}} \quad \text{.............................. (12)}$$

$$\Delta f_c = 6(\sigma_m)^{\frac{1}{2}} \quad \text{.. (13)}$$

Here, Δf_c is the strength gain while s'/d is the ratio of clear spacing between ties to the least lateral core dimension of the confined column. The clear spacing between ties was taken for this relation since it is a better representation of the unsupported concrete length between ties.

Comparison of the model predictions with experiments is given in Tab. 4. From these comparisons it is seen that the design model proposed for confinement effectiveness index and associated strength gain is successful in predicting the measured values of the experiments adopted in this study. The checking of the model was done at two levels of induced average confining stress and strength gain of the core. This aspect is significant since other available experimental data consist of only strength gain values. The average confining stress was evaluated for the first time for square confined concrete columns based on comprehensive steel strain measurements, in the experiments conducted [10].

3.4 Application of the Basic Model

The model for the confinement effectiveness and associated strength enhancement was developed based on experiments conducted by the author and results of microscopic constitutive equation based FEM analysis. To check the applicability of the model to general cases, it is prudent to apply it to predict experimental results by other researchers.

For this purpose, experimental data reported by Somes [12] was selected. These experiments have been conducted on 4 inch square columns. The lateral reinforcement used were slices of structural steel tubing of different wall thicknesses resulting in perfectly continuous ties. No longitudinal steel or cover to the lateral ties was provided. Plain concrete specimens had been cast with each batch of concrete for a set of confined columns to evaluate the unconfined strength of concrete. The yield strength of tie steel was 47 ksi (324 MPa), which is within the applicability range of the macro-model. Range of normalized spacing ratio used was 0.2-1.6, while volumetric reinforcement ratio was between 0.7% - 9.0%. Testing of the columns had been conducted at a quasi-static loading rate of about 2μ strains/sec. Three groups of specimens have been tested with each group cast in two batches. One shortcoming identified in groups A and B were the large span of time taken to complete testing of the specimens of the group which could have culminated in concrete strength varying within groups. Fabrication, detailing and testing conditions suggest very close similarity with the author's experiments except for the size of specimens and, most significantly, the measurement of lateral steel stress field. This test programme consisted of 42 confined column specimens.

The observed strength gain of group A specimens are compared with model predictions in Fig. 7. A fairly good correlation is observed between the experimental results and predictions. However, the scatter observed in the results could be due to variations in the unconfined strength estimations in the two batches that constituted this group.

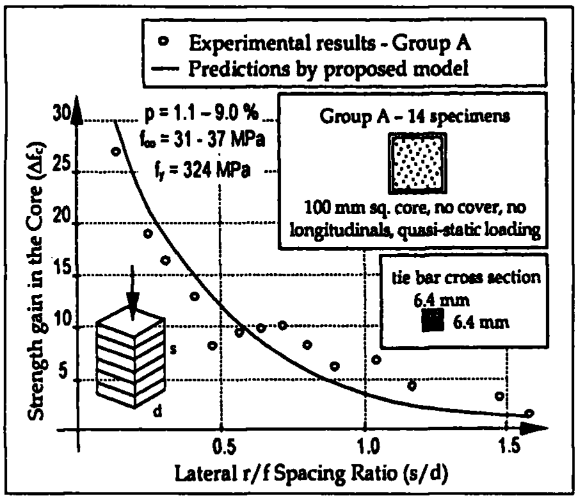

Figure 7. Strength gain prediction (Somes Group A)

Group B comparisons of strength gain are given in Fig. 8. For this group it is seen that results of columns by the first batch of concrete (s/d = 0-0.6) is quite well predicted while columns from the second batch (s/d = 0.6-1.6) show very high strength gains even as compared to experimental results for low spacing. This indicates some error in the estimation of the unconfined strength in the experiments.

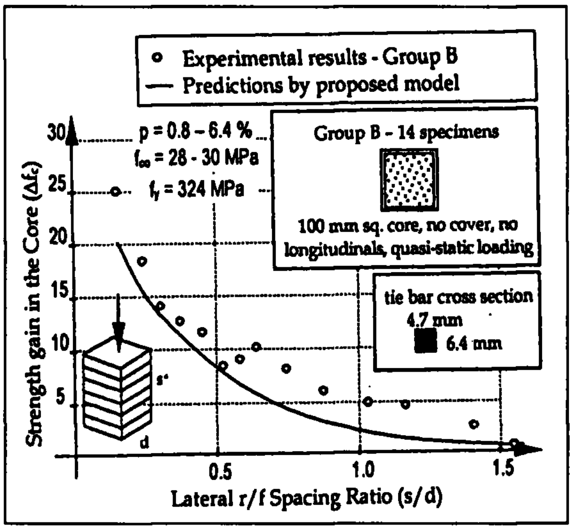

Figure 8. Strength gain prediction (Some Group B)

The group C specimens seem to be the ones tested under better control. The whole group had been tested within two days. Fig. 9, showing the strength gains and macro-model predictions indicates a very good correlation.

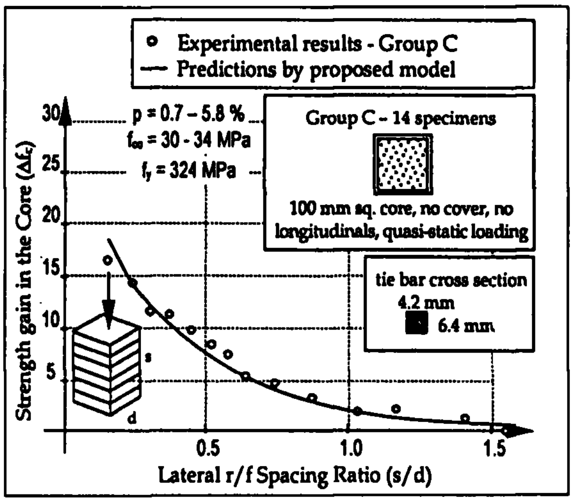

Figure 9. Strength gain prediction (Some Group C)

Confined peak strength predictions by model compared with experiments for all specimens shows good correlation, with most predictions within 10% deviation range. This implies that the proposed model is effective in predicting the strength enhancement due to confinement in the wide range of detailing parameters adopted in the study by Somes on idealized concrete cores confined by discrete lateral ties.

4. Conclusions

Based on experimental investigation on idealized square concrete columns, and micro-mechanical FEM analysis results, a "behaviour oriented model" is developed for the prediction of confinement effectiveness and strength enhancement due to passive confinement by lateral reinforcement for axi-symmetric concrete core sections. The concept of non-uniformity of lateral confining stresses, which creates weaker portions resulting in reduced confinement effectiveness, originated by the 'Elasto-plastic and fracture model' based FEM analysis was utilized in the development of this model.

Three factors, namely, *"Limit uniformity factor (K*)"*, *"Uniformity factor (K0)"* and lateral r/f spacing ratio (s/d) were introduced in the model for confinement effectiveness index. Also the contribution of flexure and shape of tie arms were incorporated in the uniformity factors by flexure/shape factor (F_r). These factors were used to account for uniformity of confinement under clear physical conditions.

Spatial average confining stress based on confinement effectiveness index is related to the strength enhancement by confinement. The second effect of lateral stress uniformity in development of strength enhancement was explicitly incorporated into the model based on conceptual minimum sectional average confining stress. This critical confining stress identified through FEM analysis was assumed at the weakest confined section, which will govern the strength of confined concrete. A unique relationship is proposed for the strength enhancement, only dependent on the above minimum confining stress. The model is based on quasi-static strain rates. Though steel yield strength in the range 300-350 MPa is specified as the applicable range for this model, BS specified steels (f_y = 250/460 MPa) are projected to be applicable. This projection is based on the rational representation of the steel yield strength in the model.

The model was applied in predicting experimental strength gains and corresponding peak strengths of laterally confined concrete cores tested by a previous researcher, and were found to satisfactorily represent individual values as well as tendencies.

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