Prediction of Shear Transfer in High-strength Concrete Crack Interfaces

P. Gatheeshgar, P. Silojan, A. J. Dammika and H. D. Yapa

Abstract: Shear behaviour of concrete is still a riddle, accurate prediction of concrete shear capacity is therefore a challenge. Shear transfers across concrete cracks primarily via aggregate interlocking based shear friction and via dowel action, amongst the former is considered as the major factor. It is found that the Contact Density Function (CDF) has the potential to quantify shear friction at cracked concrete interfaces. This function has mainly been used for normal strength concrete (NSC) (<60 MPa) and recent investigations show that it has to be modified when it is used for high strength concrete (HSC), different aggregate sizes and large crack widths. The current study focussed on the modification of the CDF to suit high-strength concrete using the experimental results of nine pre-cracked push-off specimens which comprised of compressive strength up to 90 MPa, maximum aggregate sizes of 12.5/20 mm, and comparatively large initial crack width of about 0.5 mm. The results indicate a significant prediction inaccuracy when the existing model is used for HSC specimens. However, incorporation of a modification into the formulae is found to improve the prediction accuracy for HSC reasonably.

Keywords: Aggregate interlock; Aggregate size; High-strength concrete; Shear friction

1. Introduction

Accurate prediction of the shear capacity of Reinforced Concrete (RC) structures is a challenge, as shear behaviour in RC has not been fully understood as yet. Amongst many shear models explored in the past, the path dependent shear stress transfer model is found to be quite effective [1,2]. It is an Original Contact Density (OCD) model which can deal with the complex nature of shear stress transfer in concrete by dividing the crack surface into infinitely small pieces called contact units. The directional distribution of the contact units is represented as a Contact Density Function (CDF).

For normal strength concrete (NSC), Li et al. [2] proposed a simple trigonometric formula for the CDF. For high strength concrete (HSC) applications, this CDF needs to be modified, because in NSC, cracks propagate around the coarse aggregate whereas in HSC cracks generally go through the aggregate. Hence, the HSC crack surface would be smoother than the NSC crack surface. Accordingly, Bujadham et al. [3] modified the CDF to make it an exponential function so that it can be used for HSC as well.

Moradi et al. [4] further modified the prediction approach to obtain more accurate predictions by including more parameters in the formulae. They introduced a normal distribution function (NDF) to represent the probability distribution of the contact units. Four basic governing parameters: compressive strength of concrete (f_{cu}); maximum aggregate size (G_{max}); initial crack width (w_o); and crack asperity degradation, were

incorporated into the OCD model. However, it was found that some of these modifications lack sufficient experimental verification. The current study was conducted in that context to validate the model proposed by Moradi et al. [4] for HSC using a push-off experimental series which comprised of high-strength concrete of wide range of strength and different sizes of locally available coarse aggregate.

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2. Shear Stress Transfer Concept

2.1 Contact Density Function for NSC and HSC

Li et al. [2] conducted an experimental study to explore the two dimensional projections of crack surfaces. Considering the directional distribution of constituent contact units of the crack surface shown in Figure 1, the following trigonometric function was proposed to represent the directional distribution of contact units in which θ is the contact unit inclination.

$$\Omega(\theta) = 0.5 \cos(\theta) \quad \dots (1)$$

Figure 1 - Contact units [2]

Later, it was noticed that the projection of HSC crack surface is flatter than that of NSC. So, Bujadham et al. [3] idealized the geometry of HSC crack surface using the following formula.

$$\Omega(\theta) = \frac{5}{6} e^{-21\left(\frac{\theta}{\pi}\right)^2} \quad \dots (2)$$

In a crack plane, the contact forces between the two sides of a contact unit area have to balance the external shear and normal stresses that have been applied. Hence, the integrals of the horizontal and vertical components of the contact force Z (Figure 1) balance with shear (τ) and normal (σ) stresses in the plane so that,

$$\tau = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Z \sin \theta d\theta \quad \dots (3)$$

$$\sigma = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Z \cos \theta d\theta \quad \dots (4)$$

where Z is a function of $\Omega(\theta)$. Comprehensive details of these formulae are found elsewhere [2, 4].

2.2 Incorporation of Modifications

Moradi et al. [4] incorporated several modifications into the OCD model. Their primary revision was to convert the CDF to become a zero-mean normal distribution function (NDF). The standard deviation of the NDF (σ_Ω) is governed by the standard deviation of the contact density function, maximum aggregate size (G_{max}), initial crack width (w_0), and crack asperity degradation so that,

$$\sigma_\Omega = \sigma_{F_1} \times F_2 \times F_3 \times F_4 \quad \dots (5)$$

where F_1 is the CDF (Eqn. 1 for NSC and Eqn. 2 for HSC), F_2 is a factor that represents the maximum aggregate size, F_3 is a factor relating to the initial crack width, and F_4 represents the crack roughness degradation rate.

2.2.1 σ_{F_1}

Using statistical calculations, the standard deviation (σ_{F_1}) of the CDFs for NSC (Eqn. 1) and HSC (Eqn. 2) are found to be $\sqrt{\frac{\pi^2}{4} - 2}$ and 0.48 respectively [4].

2.2.2 Factor F_2

Li et al. [2] reported that there were no noticeable differences in the two dimensional projection of crack profiles when the aggregate size was between 15 mm and 25 mm. Accordingly, Moradi et al. [4] proposed the modification factor, F_2 , to be included in the OCD when $G_{max} < 15$ mm so that,

$$F_2(G_{max}) = [1 - \psi(\eta, \alpha)] \quad \dots (6)$$

where,

$$\psi(\eta, \alpha) = \left\{ 1 - \frac{\exp(-\alpha)\eta}{1 + [\exp(-\alpha) - 1]\eta} \right\} \quad \dots (7)$$

$$\eta = \frac{G_{max}}{15} \quad \dots (8)$$

Moradi et al. while proposing that α could vary between -1 and 1 highlighted that experimental results would be necessary for its verification.

2.2.3 Factor F_3

As the initial crack width increases, the potential for contact decreases, so that shear stress transfer becomes lower. Moradi et al. [4] incorporated this idea into the OCD model using equation,

$$F_3(w) = \exp\left(\frac{1}{50w_0}\right) \quad \dots (9)$$

where w_0 is the initial crack width.

2.2.4 Factor F_4

Loading causes crack asperity degradation that resulting in the reduction of the initial standard deviation (σ_{var0}). It makes the corresponding NDF to become narrower. To cater to this, Moradi et al. [4] proposed factor,

$$F_4(\alpha) = \frac{\sigma_{var}}{\sigma_{var0}} = \exp(-\beta W^{cr}) \quad \dots (10)$$

where W^{cr} is the work spent on fracture process during loading and β controls the rate of asperity degradation. It was assumed in the OCD that during loading there was no fracture and roughness degradation and that the surface area was constant. For monotonic loading this factor would be one.

2.3 Crack width - shear Displacement Relationship

Moradi et al. [4] highlighted that the modified OCD model is effective when the initial crack width does not vary significantly with loading. Hence, to identify a model for non-constant crack widths, the following formula that expresses the relationship between shear displacement (Δ) and crack width(w) which was used in Sageseta et al.'s [5] study was used in this study.

$$w = 0.6 \Delta^{\frac{2}{3}} \quad \dots (11)$$

3. Experimental Programme

An investigation was conducted to verify proposals made by Moradi et al. [4] for HSC applications and to identify any additional necessary modifications. Nine HSC push-off specimens were cast and tested for failure. Table 1 shows the distribution of the compressive strength (61 – 90 MPa) and the maximum aggregate size (12.5 mm and 20 mm) of the specimens. The threshold strength between NSC and HSC was considered as 60 MPa [6].

3.1 Push-off Specimens

Figure 2 illustrates the main details of the specimens used. To facilitate comparisons, the dimensions of the specimens were selected to be same as those used in the work conducted by Sagaseta et al. [5].

During casting, two aluminum angle sections were used on either side of the shear plane to achieve pre-determined grooves (Figure 2). This was important to have the shear crack right along the middle of the specimen. To achieve an

effective load distribution through the expected shear plane, reinforcements were provided (Figure 2). Slip deformation across the shear plane was measured. The strain in the three T8 shear reinforcements which had a yield strength of 460 MPa was also measured using strain gauges. This was important to explore the dowel action contribution, which however is beyond the scope of this paper. Figure 3 depicts the instrument arrangement.

Table 1 - Push-off specimen parameters

Specimen	G_{max} (mm)	f_{cu} (MPa)	w_0 (mm)
SP1	12.5	61	0.43
SP2	12.5	73	0.53
SP3	12.5	75	0.56
SP4	12.5	79	0.48
SP5	12.5	85	0.60
SP6	20	71	0.56
SP7	20	79	0.50
SP8	20	80	0.74
SP9	20	90	0.52

3.2 Concrete Mixes and Casting

Ordinary Portland Cement (OPC) of strength class 42.5N was used as the binder. Supplementary cementitious materials (SCMs), silica fume and fly ash, were utilized to achieve the desired quality in concrete. A high range water reducing admixture was also added to the mixes. Crushed coarse aggregate and river sand were used as the aggregate. Three 150 mm cubes were cast as control specimens for each push-off specimen. After casting, the specimens and the cubes were cured until they were tested. All the specimens were tested after more than 28 days. Table 2 summarises the mix proportions.

3.3 Test Setup and Procedure

A 100-ton Universal Testing Machine (UTM) was used to load the specimens. The initial crack width was monitored using demec gauges (demountable mechanical strain gauges). During loading, shear slip and crack width were measured using displacement gauges.

Table 2 – Mix proportions

Specimen	Cement (kg/m ³)	Water (kg/m ³)	Coarse aggregate (kg/m ³)	Fine aggregate (kg/m ³)	Admixture (l/m ³)	Silica fume (kg/m ³)	Fly ash (kg/m ³)	f _{cu} (MPa)
SP1	502.0	216.0	944	662.5	-	-	-	61
SP2	522.5	194.0	1088	543.0	6.6	27.5	-	73
SP3	467.5	194.0	1088	543.0	7.0	55.0	27.5	75
SP4	542.8	175.1	1003	744.4	6.4	25.9	-	79
SP5	590.2	154.5	1059	681.4	7.2	-	-	85
SP6	492.0	176.0	901	799.0	5.0	25.6	-	71
SP7	508.0	165.0	1115	591.0	8.0	7.6	15.2	79
SP8	542.8	175.1	1003	744.4	6.4	25.9	-	80
SP9	590.2	154.5	1059	681.4	7.2	-	-	90

Figure 2 – Push-off specimen

Figure 3 – Instrument arrangement: (a) demec studs; (b) displacement gauges

3.3.1 Pre-cracking

All the specimens were pre-cracked deliberately along the desired failure shear plane before applying the shear force. Moradi et al. [4] found that the accuracy of the analytical prediction model reduced as the initial crack width increased. Hence, to lessen the prediction inaccuracies, the initial crack width was maintained at a high value of around 0.5 mm. Table 1 presents the initial crack widths of the specimens. Further details about the pre-cracking procedure are given in [7].

3.3.2 Testing of Push-off Specimens

The pre-cracked specimen was positioned vertically in the UTM to apply shear force. Two steel bearing plates were placed on the top and bottom faces of the specimen to ensure uniform loading. All the instruments were connected to a data logger and the initial readings were recorded. The specimen was loaded concentrically and the load was increased until the specimen failed. At each load increment, the slip along the shear plane and the crack width, were recorded using displacement gauges.

Figure 4 shows the pictures of one push-off specimen while it was being loaded and after it failed.

(a)

(b)

Figure 4 – Push-off specimen (a) at loading; (b) after failure

4. Evaluation of Push-off Test Results

To assess the proposals of Moradi et al. [4] with regard to HSC behaviour, the shear stress – displacement behaviour of the specimens was explored. The following sections discuss the main observations made.

Figures 5 (a), (b), (c), (d) and (e) illustrate the experimental and predicted shear stress – shear slip behaviours in HSC 12.5 mm aggregate specimens comprised of 61 MPa, 73 MPa, 75 MPa, 79 MPa and 85 MPa concrete, respectively. The analytical estimations include Moradi et al. predictions with three α values (-1, 0 and 1) and OCD predictions. It was observed that irrespective of the value of α , the correlation between the experimental results and the predictions is unsatisfactory.

(a)

(d)

(b)

(e)

(c)

Figure 5 - HSC specimens comprising 12.5 mm aggregate: (a) 61 MPa; (b) 73 MPa; (c) 75 MPa; (d) 79 MPa; (e) 85 MPa

The experimental and predicted shear stress - shear slip relationships for HSC 20 mm aggregate specimens are shown in Figure 6. Figures 6 (a), (b), (c) and (d) illustrate the results for specimens of 71 MPa, 79 MPa, 80 MPa and 90 MPa concrete respectively. Like the 12.5 mm aggregate specimens, all four high strength specimens also had an unsatisfactory level of prediction. Since the aggregate size was greater than 15 mm in these specimens, factor F2 (associated with α) by default is one here.

Figure 6 - HSC specimens comprising 20 mm aggregate: (a) 71 MPa; (b) 79 MPa; (c) 80 MPa; (d) 90 MPa

To address the prediction inaccuracy for the HSC specimens, the CDF pertaining to NSC (in F1) was used in the analytical model. Because it could be somewhat arbitrary to discriminate the two types of concretes based on a strength value (of 60 MPa). For a trial, the value of α was set at zero. The results obtained for the 12.5 mm and 20 mm aggregate specimens are shown in Figures 7 and 8 respectively. Interestingly, the comparisons imply that the correlation between experimental and analytical results has improved with this alteration. It can thus be concluded that the contact surfaces in the HSC specimens were more aligned with the CDF proposed for NSC than that proposed for HSC.

(a)

(d)

(b)

(e)

(c)

Figure 7 - OCD predictions associated with the NSC model for 12.5 mm aggregate specimens: (a) 61 MPa; (b) 73 MPa; (c) 75 MPa; (d) 79 MPa; (e) 85 MPa

(a)

(d)

(b)

(c)

--- Analytical model with factor
— Experiment results

Figure 8 - OCD predictions associated with the NSC model for 20 mm aggregate specimens: (a) 71 MPa; (b) 79 MPa; (c) 80 MPa; (d) 90 MPa

It has to be further noted that except the HSC modification, the other modifications (F2 and F3) of the model proposed by Moradi et al. were noticed to be effective in improving the prediction accuracy ($F4 = 1$ for the entire experiment because of using monotonic loading). For instance, Figure 9 compares the shear stress - slip behaviour of specimen SP4 with and without the other modifications. It highlights that in the absence of F2 and F3 modifications, the predictions become much more un-conservative.

Figure 9 - Influence of F2 and F3 on predictions

Overall, the shear prediction model was fairly successful for HSC push-off specimens with the NSC contact density function and with F2/F3 modifications proposed by Moradi et al. [4]. Further investigations in this area would be necessary to tune the prediction model further.

5. Conclusions

This paper presents the outcome of an on-going investigation towards prediction of shear behaviour of high-strength concrete (HSC) using a modified version of a contact density model. Based on the results obtained for nine pre-cracked push-off test specimens, the following conclusions can be drawn.

The incorporation of HSC modification (F1) in shear stress – slip predictions was not effective for both 12.5 mm and 20 mm aggregate size push-off specimens. When the shear behaviour of these specimens was predicted using the NSC contact density function, the prediction accuracy notably increased. In contrast, modifications related to maximum aggregate size and initial crack width (F2 and F3) contributed significantly to improve the prediction accuracy. Meanwhile, $\alpha = 0$ was found to be a good assumption for F2 in the modified contact density model although Moradi et al. [4] proposed it to be between -1 and 1. Further modifications should be explored in the future to enhance the prediction accuracy.

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