Aerodynamic Drag and Rolling Resistance of Three-Wheelers

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Abstract: Three-wheelers are popular in Sri Lankan roads as a convenient mode of transportation. Presently there are about 500,000 registered three-wheelers. The range of commercially available electric vehicles are limited due to their battery weight and hence used mostly as neighbourhood vehicles. Three-wheelers are used mostly as neighbourhood vehicles and hence ideal for electrification. But no successful commercial electrically powered three-wheeler is available yet. Aerodynamic drag coefficient and rolling resistance coefficient are two basic parameters needed to determine the motor capacity of an electric vehicle. Accurate values of aerodynamic drag coefficient and rolling resistance coefficient are important to calculate accurate power estimate in electric vehicle design. In this paper we present an accurate method to determine the aerodynamic drag coefficient and rolling resistance coefficient of commonly available three-wheelers in Sri Lanka.

Keywords: Aerodynamic drag coefficient, rolling resistance coefficient

1. Introduction

Aerodynamic Drag coefficient (C_d) and rolling resistance coefficient (μ) are very important basic parameters for designing energy efficient vehicles. Aerodynamic drag coefficient varies from 0.30 to 0.35 for modern cars, from 0.33 to 0.35 for modern vans and from 0.42 to 0.46 for modern pick-ups [1]. These parameters are obtained by carrying out a coast-down test where vehicle is driven to a reasonable high speed, put into neutral gear if possible, and left to roll unrestricted to a complete stop on a levelled dry asphalt-covered road[3]. Then with the aid of the speed-time graph, mean velocity and mean acceleration at a relatively high speed (50 kmh⁻¹) and relatively low speed (10 kmh⁻¹) are estimated to determine the above two parameters. Although this is a relatively simple method, it does not give accurate results for light weight vehicles like three-wheelers because for light weight vehicles, especially at low speeds, imperfect road conditions create significant speed variations. This paper presents an accurate methodology for determining C_d and μ for low weight vehicles such as three-wheelers.

2. Methodology

The aerodynamic drag coefficient and rolling resistance creates forces F_d and F_r

respectively to decelerate the vehicle as follows:

$$F_d = 0.5\rho C_d A(v - v_0)^2 \quad \text{----- (1)}$$

Where,

ρ - Air density (kgm⁻³)

C_d - Drag coefficient

A - Frontal projected area of the vehicle (m²)

v - Vehicle speed (ms⁻¹)

v_0 - Wind Speed (ms⁻¹)

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$$F_r = \mu mg \cos(\alpha) \quad \text{----- (2)}$$

Where,

μ - Rolling resistance

α - Inclination angle (degree)

$\square$ - Weight of the vehicle (kg)

Hence, following differential equation will describe the behaviour of the vehicle during coast-down test at zero wind speed on a flat road.

$$m \frac{dv}{dt} = 0.5 \rho C_d A v^2 + \mu mg \quad \text{----- (3)}$$

Then by defining two constants C_1 and C_2 ,

$$C_1 = \frac{m}{0.5 \rho C_d A} \quad \text{----- (4)}$$

$$C_2 = \sqrt{\frac{\mu mg}{0.5 \rho C_d A}} \quad \text{----- (5)}$$

The differential equation can be simplified as

$$C_1 \frac{dv}{dt} = v^2 + C_2^2 \quad \text{----- (6)}$$

The general solution to the above differential equation can be expressed as

$$v = C_2 \tan \left(K - \left(\frac{C_2}{C_1} \right) t \right) \quad \text{----- (7)}$$

K is a constant

Taking the boundary condition $v=0$ at $t = t_e$

t_e = End time

$$K = \left(\frac{C_2}{C_1} \right) t_e \quad \text{----- (8)}$$

The reduction in the velocity of the vehicle during the coast-down test can finally be described as

$$v = C_2 \tan \left(\left(\frac{C_2}{C_1} \right) (t_e - t) \right) \quad \text{----- (9)}$$

Substituting C_1 and C_2 values to Equation 9

$$v = \left(\sqrt{\frac{\mu mg}{0.5 \rho C_d A}} \right) \cdot \tan \left(\left(\frac{\left(\sqrt{\frac{\mu mg}{0.5 \rho C_d A}} \right)}{\left(\frac{m}{0.5 \rho C_d A} \right)} \right) (t_e - t) \right) \quad \text{----- (10)}$$

C_d and μ are only two unknowns in this equation.

3. Parameter Estimation and Verification for a Flat-Even Road

The following data is applicable for a commonly used three-wheeler:

Mass (m): 500 kg

Frontal projected area ($\square$): 1.595 m²

Taking Air Density (ρ) as 1.1613 kgm⁻³ at an elevation of zero meters and ambient temperature of 30 °C, C_1 and C_2 can be expressed as

$$C_1 = \frac{539.8770}{C_d} \quad \text{----- (11)}$$

$$C_2 = 72.7366 \cdot \sqrt{\frac{\mu}{C_d}} \quad \text{----- (12)}$$

Hence, the three-wheeler speed during the coast-down test can be given as

$$v = 72.7366 \sqrt{\frac{\mu}{C_d}} \cdot \tan \left((0.1347 \sqrt{C_d \mu}) (t_e - t) \right) \quad \text{----- (13)}$$

With these non-linear relationships, extraction of parameters C_d and μ from a measured data set is very difficult. The best way to extract parameters is by selecting the best curve from a set of theoretical plots generated for a number of C_d and μ values giving the minimum Mean Square Error (MSE) and the largest correlation coefficient with the measured data set. Figure 1 depicts the graph for a typical measurement carried out using an electronic data logger develop by us for a three wheeler with the best fitting theoretical graph having MSE of 0.7562 and correlation coefficient of 0.9992. According to results tabulated in Table 1, the drag coefficient of the vehicle is 0.4080 and the rolling resistance coefficient is 0.0169. The values obtained by

the method described in [2] are 0.4702 and 0.0166 respectively. The MSE for the above result is 1.9485 and the correlation coefficient is 0.9987. The results obtained from the two methods show a significant agreement for an even road compared to results presented in Section 4 for an uneven road.

	Proposed Method		Method described in [2]	
	C_d	μ	C_d	μ
	0.4080	0.0169	0.4702	0.0166
Correlation coefficient	0.9992		0.9987	
Mean square error	0.7562		1.9485	

Table 1- C_d and μ values estimated from two independent methods for a flat-even road

4. Parameter Estimation for a Flat-Uneven Road

Three-wheeler is a light vehicle and surface irregularities of the road affect vehicle speed in a significant manner. In the method described in [2], parameters are estimated using the mean velocity and mean acceleration at a relatively high speed point and relatively low speed point and hence the surface irregularities introduce significant errors. Table 2 shows estimated C_d and μ values using proposed method and method described in [2] for four different attempts on the same road. A large variation is observed for C_d and μ obtained from the method described in [2] whereas less variation is observed for the values estimated from proposed method.

Figure 1 - Actual and theoretical graphs for a flat-even road

Table 2 - Optimum C_d and μ values for a flat-uneven road

Attempt	Proposed Method				Method Described in [2]			
	C_d	μ	Correlation Coefficient	Mean square error	C_d	μ	Correlation Coefficient	Mean square error
1 st	0.5800	0.02680	0.9985	0.1942	0.5758	0.0204	0.9983	10.7078
2 nd	0.5300	0.02800	0.9989	0.1562	0.6027	0.0218	0.9987	6.6676
3 rd	0.5500	0.0300	0.9983	0.1715	0.4448	0.0334	0.9985	1.1438
4 th	0.5700	0.0270	0.9988	0.2303	0.4125	0.0300	0.9984	1.0599
Average	0.5575	0.0279			0.5089	0.0264		
Standard Deviation	0.0192	0.0013			0.0817	0.0055

Figure 2 - Actual and theoretical graphs for a flat-uneven road (2nd attempt)

5. Conclusions

Surface irregularities of the roads tend to change the speed of light weight vehicles such as three-wheelers during coast-down test. These speed variations create significant changes to the calculation of aerodynamic drag coefficient and rolling resistance by the methods presented in literature. For example, using the method described in [2], the aerodynamic drag coefficient was found to be 0.5089 and rolling resistance was 0.0264 with a standard deviation of 0.0817 and 0.0055 respectively for our test vehicle. For the same test, using the new method in this paper, we have obtained aerodynamic drag coefficient and rolling resistance as 0.5575 and 0.0279 with a standard deviation of 0.0192, 0.0013 respectively. Therefore, the method presented in this paper is better suited to determine aerodynamic drag coefficient and rolling resistance of light weight vehicles such as three-wheelers.

References

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Appendix A: Example empirical calculation using method described in [2]

Table A1 - Case example for empirical calculation of C_d and μ

Sample pair 1		Sample pair 2	
t_a, v_a	(0 s, 21.95 ms ⁻¹)	t_a, v_a	(74 s, 2.16 ms ⁻¹)
t_b, v_b	(4 s, 19.78 ms ⁻¹)	t_b, v_b	(79 s, 1.33 ms ⁻¹)
Mean Velocity $v_1 = \frac{v_a + v_b}{2}$	$v_1 = 20.865 \text{ ms}^{-1}$	Mean Velocity $v_2 = \frac{v_a + v_b}{2}$	$v_2 = 1.745 \text{ ms}^{-1}$
Mean Deceleration $a_1 = \frac{v_b - v_a}{t_b - t_a}$	$a_1 = -0.5425 \text{ ms}^{-2}$	Mean Deceleration $a_2 = \frac{v_b - v_a}{t_b - t_a}$	$a_2 = -0.166 \text{ ms}^{-2}$
Aerodynamic Drag Coefficient	$C_d = \frac{2m(a_2 - a_1)}{\rho A(v_1^2 - v_2^2)} = 0.4702$		
Rolling Resistance Coefficient	$\mu = \frac{(a_1 v_2^2 - a_2 v_1^2)}{g(v_1^2 - v_2^2)} = 0.01665$		

Appendix B: Coast-down test data for a three wheeler

Table B1 – Measured Velocity for a Flat-Even Road

Time(s)	Velocity(ms ⁻¹)	Time(s)	Velocity(ms ⁻¹)	Time(s)	Velocity(ms ⁻¹)
0	21.95	30	10.64	60	4.66
1	21.61	31	10.64	61	4.32
2	20.95	32	10.47	62	4.16
3	20.62	33	10.31	63	3.99
4	19.78	34	9.98	64	3.82
5	19.29	35	9.81	65	3.49
6	18.95	36	9.64	66	3.49
7	18.45	37	9.48	67	3.16
8	17.96	38	9.31	68	3.16
9	17.79	39	9.31	69	2.99
10	17.29	40	9.14	70	2.83
11	17.12	41	8.81	71	2.49
12	16.63	42	8.65	72	2.33
13	16.46	43	8.31	73	2.33
14	15.96	44	8.15	74	2.16
15	15.79	45	7.98	75	2.00
16	15.30	46	7.81	76	1.83
17	15.13	47	7.48	77	1.50
18	14.63	48	7.32	78	1.33
19	14.30	49	6.98	79	1.33
20	13.97	50	6.82	80	1.00
21	13.63	51	6.65	81	1.00
22	13.30	52	6.32	82	0.67
23	12.47	53	5.99	83	0.67
24	12.47	54	5.82	84	0.50
25	11.97	55	5.65	85	0.33
26	11.47	56	5.49	86	0.17
27	11.31	57	5.15	87	0.00
28	11.14	58	5.15		
29	10.81	59	4.82

Table B2 – Measured velocity for a Flat-Uneven Road

Time (s)	Attempt 1 Velocity (ms ⁻¹)	Attempt 2 Velocity (ms ⁻¹)	Attempt 3 Velocity (ms ⁻¹)	Attempt 4 Velocity (ms ⁻¹)	Time (s)	Attempt 1 Velocity (ms ⁻¹)	Attempt 2 Velocity (ms ⁻¹)	Attempt 3 Velocity (ms ⁻¹)	Attempt 4 Velocity (ms ⁻¹)
0	21.39	21.17	20.39	20.73	29	7.09	7.20	5.99	6.98
1	20.84	20.73	19.84	20.17	30	7.20	6.54	5.65	6.65
2	20.28	19.95	19.17	19.62	31	6.54	6.10	5.43	6.76
3	19.51	19.17	18.51	18.84	32	6.10	5.87	5.10	6.21
4	18.95	18.51	17.73	18.29	33	5.87	5.43	4.77	5.87
5	18.29	17.84	17.29	17.84	34	5.43	5.21	4.43	5.43
6	17.84	17.29	16.74	17.29	35	5.21	4.88	4.21	5.32
7	17.29	16.85	16.29	16.85	36	4.88	4.66	3.66	4.88
8	16.85	16.40	15.74	16.40	37	4.77	4.43	3.66	4.66
9	16.40	15.85	15.30	15.96	38	4.32	4.32	3.77	4.32
10	15.85	15.52	14.85	15.41	39	4.10	3.77	2.99	4.43
11	15.41	14.96	14.41	14.96	40	3.99	3.99	2.77	3.77
12	14.96	14.52	13.97	14.52	41	3.77	3.33	2.44	3.66
13	14.41	13.97	13.52	13.97	42	3.55	3.21	2.44	3.44
14	14.08	13.63	13.08	13.52	43	2.99	2.88	2.00	2.88
15	13.63	13.08	12.64	13.08	44	2.88	2.66	1.77	2.77
16	13.19	12.75	12.19	12.64	45	2.66	2.33	1.66	2.55
17	12.75	12.30	11.86	12.08	46	2.44	2.11	1.11	2.22
18	12.30	11.86	11.31	11.64	47	2.22	2.00	1.11	1.88
19	11.75	11.31	10.86	10.97	48	1.77	1.66	0.78	1.66
20	11.31	10.97	10.42	10.53	49	1.44	1.33	0.33	1.44
21	10.86	10.42	9.98	9.86	50	1.55	1.33	0.11	1.22
22	10.42	9.98	9.31	9.31	51	1.11	0.89	0.00	0.89
23	9.75	9.53	8.87	8.98	52	0.78	0.78	-	0.78
24	9.09	8.87	8.20	8.53	53	0.44	0.55	-	0.44
25	8.76	8.98	7.87	8.09	54	0.33	0.11	-	0.33
26	8.31	8.20	7.76	7.76	55	0.11	0.00	-	0.11
27	8.42	7.98	6.76	8.31	56	0.00	-	-	0.00
28	7.32	7.65	7.20	7.32					

Table B3 – Selected data sets to calculate C_d and μ for flat-uneven road by method in [2]

	Sample pair 1		Sample pair 2	
1 st Attempt	t_a, v_a	(0 s, 21.39 ms ⁻¹)	t_a, v_a	(43 s, 2.99 ms ⁻¹)
	t_b, v_b	(5 s, 18.29 ms ⁻¹)	t_b, v_b	(50 s, 1.55 ms ⁻¹)
2 nd Attempt	t_a, v_a	(0 s, 21.17 ms ⁻¹)	t_a, v_a	(41 s, 3.33 ms ⁻¹)
	t_b, v_b	(7 s, 16.85 ms ⁻¹)	t_b, v_b	(47 s, 2.00ms ⁻¹)
3 rd Attempt	t_a, v_a	(0 s, 20.39 ms ⁻¹)	t_a, v_a	(38 s, 3.77 ms ⁻¹)
	t_b, v_b	(5 s, 17.29 ms ⁻¹)	t_b, v_b	(46 s, 1.11 ms ⁻¹)
4 th Attempt	t_a, v_a	(0 s, 20.73 ms ⁻¹)	t_a, v_a	(32 s, 6.21 ms ⁻¹)
	t_b, v_b	(5 s, 17.84 ms ⁻¹)	t_b, v_b	(38 s, 4.32 ms ⁻¹)