Analysis of the Prestress Transfer Bond Problem Incorporating Cohesive Cracking of Concrete

I R A Weerasekera

Abstract : Accurate prediction of prestress transfer bond is a difficult task. A sound theoretical basis is lacking to complement the experimental evidence available in the literature. An analysis procedure treating cracked concrete as an anisotropic material is presented here. Using this approach, improved transfer length predictions can be made. An analytical solution for the formulated non-linear non-homogeneous differential equation is not feasible. Hence a numerical solution obtained using a modified fourth order Runge- Kutta method as an initial value problem is successfully applied. The numerical results were found to be accurate and the solution technique efficient.

Keywords: Transfer bond, pretension, cracked concrete, uncracked concrete, isotropic analysis, anisotropic analysis, cohesive cracking, and transfer length.

1. Introduction

Pretensioned prestressed concrete beams have been in use for more than 70 years. Early practice made use of 2 mm wire but recent trends are towards use of 13, 15 and 16 mm diameter strand in such pretensioned members. There are several problems associated with prediction of transmission of prestress by bond. Experimental results on bond as related to large strand sizes are limited. An extensive experimental programme is also expensive. The absence of a valid theory to complement experiments has stifled the search for rational solutions to the bond problem.

In this paper a detailed procedure is presented to predict bond development. In this approach the complications resulting from the complex material properties of cracked concrete are resolved.

2. Problem formulation

2.1 General

To make an analysis possible, it is necessary to make some simplifying assumptions. Steel is treated as a solid cylinder and concrete as a hollow cylinder with a radius equal to the lesser of the smaller concrete cover or one-half the distance to the closest adjacent reinforcing. The outer surface of the notional concrete cylinder is assumed to behave as a free surface. The physical dimensions are shown in Fig. 1, where the radius of the steel cylinder is equal to r_o , the radius of the unstressed steel. The concrete cylinder has as outer radius of C and an inner radius of r_i , which is the radius of the

stressed steel because the concrete is cast when the tendon is in its stressed state. Due to Poisson's ratio, r_i is always less than r_o . At prestress transfer the steel shortens and swells so a pressure p develops at the interface. The basic principle was already understood and described by Hoyer and Friedrich [1], but in their analysis an unconfined situation has been adopted. The resulting equation does not agree very well with test results.

An isolated steel part from the sliced element is shown in Fig. 2. The change of force in the steel is based on the assumption that a constant coefficient of friction can be assumed such that

$$A_b \Delta f_{pz} = \mu (d_b \Delta z \pi) p \quad (1)$$

Using thick cylinder theory as illustrated in 4Fig.1, expressions for stresses, strains and displacements can be developed by ensuring equilibrium and compatibility, and imposing boundary conditions and constitutive relationships that represent the material behaviour.

The equilibrium equations and strain-displacement equations are identical for both isotropic and anisotropic analysis.

Eng. Dr. I R A Weerasekera is a Senior Lecturer at the Department of Civil Engineering, University of Moratuwa, Moratuwa, Sri Lanka. He received his B.Sc (Eng.) from University of Moratuwa and PhD from The University of Calgary, Calgary, Alberta, Canada. He is also a Chartered Civil Engineer. His research interests are reinforced and prestressed concrete structures, computer aided analysis and design and bridge engineering.

2.2 Equilibrium Equation

For the element shown in Fig. 3, equilibrium of forces in the radial direction, r , yields

$$\sigma_r + \frac{\partial \sigma_r}{\partial r} r - \sigma_\theta + \frac{\partial \tau_{rz}}{\partial z} r = 0 \quad (2)$$

In the longitudinal direction all variables are assumed to be independent of z , within the finite length Δz . Therefore the above equation reduces to

$$\sigma_r - \sigma_\theta + \frac{d\sigma_r}{dr} r = 0 \quad (3)$$

2.3 Strain-displacement Equations

These equations can be derived directly, using the element shown in Fig. 3, from considering a radial displacement of u for the inner radius and $(u + du)$ for the outer radius.

$$\epsilon_r = \frac{du}{dr} \quad (4)$$

$$\epsilon_\theta = \frac{u}{r} \quad (5)$$

2.4 Compatibility

The basic requirement to be fulfilled is the compatibility of displacement at the steel-concrete interface given by

$$r_o + u_o = u_j + r_j \quad (6)$$

where

u_o - radial displacement of outer surface of steel cylinder

u_j - radial displacement of inner surface of concrete cylinder

2.5 Boundary condition

For the concrete cylinder the assumption of an outer free surface corresponds to a radial stress, σ_r , equal to zero when $r = c$. This condition applies for all the cases discussed in this paper.

2.6 Constitutive Relations

The constitutive relations can only be meaningful if the material behaviour is represented adequately. In modelling the behaviour, the two materials were initially assumed to behave as homogeneous isotropic, linear elastic materials. These assumptions are valid for prestressing steel at the transfer stage as the steel is not stressed beyond its elastic limit. For concrete, however, the above assumptions are justifiable only if both tensile

and the compressive stresses remain within the linear range.

To examine the state of stress and the validity of the assumptions, an isotropic elastic analysis was performed. An element in the form of a slice of thickness Δz is taken at a distance z from the end. The stresses in the longitudinal direction are f_{pz} and f_{cz} for the stresses in prestressing steel and concrete respectively. (A detailed derivation of the analysis and relevant equations for stresses, strain and displacement are found by Weerasekera [2]). The stresses and strains at every location must satisfy the basic stress strain equation:

$$\begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 - \nu & -\nu & -\nu \\ -\nu & 1 - \nu & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} \quad (7)$$

2.7 Radial and circumferential stresses for uncracked concrete

Eq. (7) can be solved in terms of ϵ_r , ϵ_θ and ϵ_z to find the stresses σ_r and σ_θ . The solution is slightly different than solution in standard text books because it is based on two strains and one stress.

Radial Stress

$$\sigma_r = \frac{E}{(1 - \nu^2)} \{ \epsilon_r + \nu \epsilon_\theta \} + \frac{\nu(1 + \nu) \sigma_z}{(1 - \nu^2)} \quad (8)$$

Circumferential Stress

$$\sigma_\theta = \frac{E}{(1 - \nu^2)} \{ \epsilon_\theta + \nu \epsilon_r \} + \frac{\nu(1 + \nu) \sigma_z}{(1 - \nu^2)} \quad (9)$$

2.8 Analytical solution for u

Using Eq. (4), (5), (8) and (9) in equilibrium Eq. (3), the usual differential equation can be obtained.

$$\frac{d}{dr} \left\{ \frac{1}{r} \left[\frac{d}{dr} (ur) \right] \right\} = 0 \quad (10)$$

An analytical solution is available for this differential equation

$$u = c_1 r + \frac{c_2}{r} \quad (11)$$

Using these Eq. in (4) and (5) and then in Eq. (8) and (9), it is possible to show

$$\sigma_r = E \left\{ \frac{c_1}{(1-\nu)} - \frac{c_2}{r^2(1+\nu)} \right\} + \frac{\nu \sigma_z}{(1-\nu)} \quad (12)$$

$$\sigma_\theta = E \left\{ \frac{c_1}{(1-\nu)} + \frac{c_2}{r^2(1+\nu)} \right\} + \frac{\nu \sigma_z}{(1-\nu)} \quad (13)$$

The constants c_1 and c_2 are determined using the specific boundary conditions.

3. General equations for stresses and displacements for uncracked regions

3.1 CASE (a) Solid Steel Cylinder

When $r = 0$, $u = 0$, Therefore for all r

$$\sigma_r = \sigma_\theta = f_r, \quad (14)$$

Using the expression for constants in Eq. (11), displacement can be determined as

$$u = \left[\frac{f_r(1-\nu) - \nu \sigma_z}{E} \right] r \quad (15)$$

3.2 CASE (b) Hollow Uncracked Concrete Cylinder

When $r = R$, $\sigma_r = f_r$ and when $r = c$, $\sigma_r = 0$. Therefore the following equations can be obtained.

$$\sigma_r = \frac{f_r(1/c^2 - 1/r^2)}{(1/c^2 - 1/R^2)} \quad (16)$$

$$\sigma_\theta = \frac{f_r(1/c^2 + 1/r^2)}{(1/c^2 - 1/R^2)} \quad (17)$$

$$u = \frac{f_r r}{E(1/c^2 - 1/R^2)} \left[\frac{(1-\nu)}{c^2} + \frac{(1+\nu)}{r^2} \right] - \frac{\nu \sigma_z r}{E} \quad (18)$$

$$\frac{du}{dr} = \frac{f_r}{E(1/c^2 - 1/R^2)} \left[\frac{(1-\nu)}{c^2} - \frac{(1+\nu)}{r^2} \right] - \frac{\nu \sigma_z}{E} \quad (19)$$

3.3 CASE (c) Uncracked portion of Hollow Partially Cracked Concrete Cylinder

When $r = R$, $\sigma_\theta = f_\theta$ and when $r = c$, $\sigma_r = 0$. Therefore we obtain

$$\sigma_r = \frac{f_\theta(1/c^2 - 1/r^2)}{(1/c^2 + 1/R^2)} \quad (20)$$

$$\sigma_\theta = \frac{f_\theta(1/c^2 + 1/r^2)}{(1/c^2 + 1/R^2)} \quad (21)$$

$$u = \frac{f_\theta r}{E(1/c^2 + 1/R^2)} \left[\frac{(1-\nu)}{c^2} + \frac{(1+\nu)}{r^2} \right] - \frac{\nu \sigma_z r}{E} \quad (22)$$

$$\frac{du}{dr} = \frac{f_\theta}{E(1/c^2 + 1/R^2)} \left[\frac{(1-\nu)}{c^2} - \frac{(1+\nu)}{r^2} \right] - \frac{\nu \sigma_z}{E} \quad (23)$$

3.4 Equation for Displacement and Contact Pressure

For steel cylinder, the displacement at its outer surface can be written from Eq. (15)

$$u_o = \frac{-p(1-\nu_p)r_o}{E_p} - \nu_p \frac{f_{pz}}{E_p} r_o \quad (24)$$

For the uncracked concrete cylinder, the displacement at its inner surface is given by Eq.(18) as

$$u_j = \frac{-pr_j}{E_c \left(\frac{1}{c^2} - \frac{1}{r_j^2} \right)} \left[\left(\frac{1-\nu_c}{c^2} \right) + \left(\frac{1+\nu_c}{r_j^2} \right) \right] - \frac{\nu_c f_{cz}}{E_c} r_j \quad (25)$$

Using these displacements in the compatibility Eq. (6) an expression can be obtained for the interface pressure as:

$$p = \frac{r_o(1-\nu_p f_{pz}/E_p) - r_j(1-\nu_c f_{cz}/E_c)}{(1-\nu_p)r_o/E_p + \left[\nu_c + (c^2 + r_j^2)/(c^2 - r_j^2) \right] r_j/E_c} \quad (26)$$

Using this value of p as the stress f_r in Eq. (16) and (17) it is possible to obtain the stresses in the both radial and circumferential directions. The theoretical analysis shows that over much of the transfer length the tensile stresses in the circumferential direction adjacent to the steel exceed the concrete tensile strength. Therefore it is clear that the concrete must crack in the radial direction and the behaviour becomes anisotropic. Three possible situations need to be considered along the transfer length. These will be referred to as uncracked (case I), partially cracked (case II) and fully cracked (case III) as shown in Fig. 4.

Tepfers [3] used a simple model for the identification of cracking in reinforced

concrete but the effect of circumferential stresses in the cracked region was not taken into consideration.

4. Anisotropic elastic analysis for concrete with radial cracking

4.1 Constitutive Relations

The equilibrium equations and strain - displacement equations in Eq. (3) to (6) are identical for both the isotropic and anisotropic analysis. However, the constitutive relations differ. The conditions for the anisotropic state can be written in the following general form.

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{Bmatrix} = \begin{bmatrix} 1/E_r & -\nu_{\theta r}/E_\theta & -\nu_{zr}/E_z \\ -\nu_{r\theta}/E_r & 1/E_\theta & -\nu_{z\theta}/E_z \\ -\nu_{rz}/E_r & -\nu_{\theta z}/E_\theta & 1/E_z \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} \quad (27)$$

For a stratified material, with θ as the direction normal to the plane of stratification ($r-z$), the anisotropic elastic properties reduce to the following:

$$E_r = E_z = E_c \quad (28)$$

E_θ is a function of r that can be chosen to fit the particular conditions.

$$E_\theta = g(r) \quad (29)$$

$$\nu_{rz} = \nu_{zr} = \nu_{z\theta} = \nu_{r\theta} = \nu_c \quad (30)$$

$$\nu_{\theta r} = \nu_{\theta z} = \nu_c \frac{E_\theta}{E_c} \quad (31)$$

It is instructive to examine a region in the vicinity of a crack presented by Hillerborg [4]. This type of cracking subsequently termed as "cohesive cracking" by Weerasekera and Loov [5], continues to transmit some tension across the crack. The length of the cracked zone (L_c) is defined as the distance measured from the effective crack tip to the surface of the steel. To represent the degree of cracking, two parameters, the reduced maximum tensile strength (f_{tr}) and the reduced effective circumferential modulus are important.

The reduced modulus of elasticity in the θ direction (E_θ) is determined using a stress crack-width curve based on experiment [2],

successfully matched this curve as rectangular hyperbola for this analysis, by utilizing the experimental result [6]. This can be done without referring to a particular gauge length. For this particular problem it was assumed that three cracks would be sufficient to allow the tendon to expand.

Substituting these values into Eq. (27), the following equation can be obtained.

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{Bmatrix} = \frac{1}{E_c} \begin{bmatrix} 1 & -\nu_c & -\nu_c \\ -\nu_c & \frac{E_c}{E_\theta} & -\nu_c \\ -\nu_c & -\nu_c & 1 \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} \quad (32)$$

4.2 Radial and Circumferential Stresses for Cracked Concrete

Solving Eq. (32) in terms of strain ε_r and ε_θ and stresses ε_z , stresses can be expressed as

Radial Stress

$$\sigma_r = \frac{E_c}{(E_c - \nu_c^2 E_\theta)} [E_c \varepsilon_r + \nu_c E_\theta \varepsilon_\theta] + \frac{\nu_c (E_c + \nu_c E_\theta) \sigma_z}{(E_c - \nu_c^2 E_\theta)} \quad (33)$$

Circumferential stress

$$\sigma_\theta = \frac{E_c E_\theta}{(E_c - \nu_c^2 E_\theta)} [\nu_c \varepsilon_r + \varepsilon_\theta] + \frac{\nu_c E_\theta (1 + \nu_c) \sigma_z}{(E_c - \nu_c^2 E_\theta)} \quad (34)$$

4.3 Governing Equation for Radial Displacement

The solution for u using Eq. (33) and (34), combined with the equilibrium Eq. (3) the following differential equation can be obtained.

$$\begin{aligned} r E_c^2 [E_c - \nu_c^2 E_\theta] \frac{d^2 u}{dr^2} + E_c^2 [E_c + r \nu_c^2 E'_\theta - \nu_c^2 E_\theta] \frac{du}{dr} \\ + E_c [r E_c \nu_c E'_\theta - E_\theta (E_c - \nu_c^2 E_\theta)] \frac{u}{r} \\ + [(E_c - \nu_c^2 E_\theta)(E_c - E_\theta) + r \nu_c E_c (1 + \nu_c) E'_\theta] \sigma_z \nu_c = 0 \end{aligned} \quad (35)$$

Eq. (35) above is a 2nd order nonlinear non-homogeneous equation. Therefore an analytical solution is not feasible so this equation must be solved using a numerical procedure.

5. Numerical solution procedure

5.1 (R-K)⁴ Method

The solution is slightly different than the solutions found in standard text books such as reference [7], because it is based on radial increments of r .

To use the numerical technique, the 4th order Runge- Kutta (R-K)⁴ method [7] (as an initial value problem), Eq.(35) is rearranged to the following form:

$$u'' = f(u', u, r) \quad (36)$$

$$\text{ie. } \frac{d^2 u}{dr^2} = \delta \left\{ \alpha \frac{du}{dr} + \beta \frac{u}{r} + \gamma \right\} \quad (37)$$

Where

$$\delta = 1 / \left[r E_c^2 (E_c - \nu_c^2 E_\theta) \right] \quad (38)$$

$$\alpha = E_c^2 \left[\nu_c^2 E_\theta - E_c - r \nu_c^2 E'_\theta \right] \quad (39)$$

$$\beta = E_c \left[E_\theta (E_c - \nu_c^2 E_\theta) - r E_c \nu_c E'_\theta \right] \quad (40)$$

$$\gamma = \left[(E_c - \nu_c^2 E_\theta) (E_\theta - E_c) - r \nu_c E_c (1 + \nu_c) E'_\theta \right] \nu_c \sigma_z \quad (41)$$

5.2 Auxiliary Terms

If initial values are u'_n, u_n, r_n and the step size is t , then the following auxiliary terms can be computed in sequence:

$$A_n = \frac{1}{2} t f(u'_n, u_n, r_n) \quad (42)$$

$$\beta_n = \frac{1}{2} t \left(u'_n + \frac{1}{2} A_n \right) \quad (43)$$

$$B_n = \frac{1}{2} t f \left(u'_n + A_n, u_n + \beta_n, r_n + \frac{1}{2} t \right) \quad (44)$$

$$C_n = \frac{1}{2} t f \left(u'_n + B_n, u_n + \beta_n, r_n + \frac{1}{2} t \right) \quad (45)$$

$$\delta_n = t(u'_n + C_n) \quad (46)$$

$$D_n = \frac{1}{2} t f(u'_n + 2C_n, u_n + \delta_n, r_n + t) \quad (47)$$

$$K_n = \frac{1}{3} (A_n + B_n + C_n) \quad (48)$$

$$K_n^* = \frac{1}{3} (A_n + 2B_n + 2C_n + D_n) \quad (49)$$

Updated values after the increment
The values at the end of the step are

$$u'_{n+1} = u'_n + K_n^* \quad (50)$$

$$u_{n+1} = u_n + t(u'_n + K_n) \quad (51)$$

$$r_{n+1} = r_n + t \quad (52)$$

5.3 Application to Partially Cracked Region

Initially condition can be obtained by choosing L_c (Cracked length for partially cracked case) or f_{tr} (for maximum reduced tensile strength for fully cracked case) and using in Eq. (20) to get σ_r , Eq. (21) to get σ_θ , Eq. (22) to get u . By repeating the procedure with $u'_{n+1}, u_{n+1}, r_{n+1}$ as new initial values and so on, with a suitable number of steps these quantities at the inner radius can be found. The step by step analysis can be carried out for increments of Δz from the free end until the steel stress reaches f_{pi} the initial prestress after transfer where the bond stress is zero. In this process the stress in prestressing steel is transferred to the concrete including the incremental change that occurs within the finite length Δz .

6. Calibration

In order to calibrate the theoretical analysis an experimental study was performed to establish the coefficient of friction of the interface. The properties and parameters of three groups of test beams [2] are shown in Table 1. Each group of strain measurement was made up of eight or twelve readings from the two sides and two ends of the two or three beams in each group.

6.1 Determination of Transfer Length

The estimation of the transfer length using concrete strains has to be based on some objective criterion because the full prestress is obtained only after a long gradual transition. Unfortunately many researchers have not been explicit in defining this length. In this paper the following approach to define the transfer length has been made. The strain corresponding to the shrinkage strain plus 90% of the increased strain is determined. Then a line is drawn through the shrinkage strain at the end and the 90% intersection. This line is then extrapolated to intersect the maximum strain. The distance from the end to this intersection point is considered to be the transfer length, as shown in Fig. 5. For practical application this method defines the transfer length appropriately.

6.2 Comparison of Theoretical with Experimental

Measured and theoretical transfer lengths for three groups of beams with clearly defined covers are compared in Table 2. It was observed that use of a coefficient of friction of 0.75 along with an assumed concrete tensile strength of $0.6\sqrt{f'_{ci}}$ gave theoretical results that closely matched the experimental measurements. The theoretical and experimental strain curves for typical cases are shown in Fig. 6, 7 and 8. It can be seen that these curves match very well.

7. Predicted prestress build-up curves

After calibration with the test results the proposed numerical procedure was used in a comprehensive study, to generate data to predict the influence of other variables. The data used are given in Table 3. The influence of longitudinal stress, f_{ce} , in the concrete was found to be negligible. The range of values chosen for the variables is typical of that commonly used in practice. The conclusions drawn from the analysis of the data are limited to the following ranges of values.

$$20 \leq c_y \leq 40 \text{ mm}$$

$$9.53 \leq d_b \leq 15.24 \text{ mm}$$

$$20 \leq f'_{ci} \leq 40 \text{ MPa}$$

$$900 \leq f_{pi} \leq 1300 \text{ MPa}$$

Variations of the predicted prestress profile are shown in Fig. 9 and 10 for 15.24 mm strands. For 9.53 mm strands the spread of curves is very much less. The analysis is very sensitive to small incremental changes in the steel diameter. Irregularities or surface roughness, which might produce an effective increase of diameter during slip are likely to be of great importance. A value of μ equal to 0.75 matches test results. This calibration is used throughout the analysis.

7.1 Average Cover

To define the concrete cylinder, an average cover is required when multiple tendons or variable covers are present. The most appropriate value for the average cover is not obvious at this time. However it is clear that the average cover is between the minimum and maximum covers.

However, comparison of results with those in the literature is difficult as the average cover

cannot be readily established when the covers vary on different sides. This is especially difficult when there are multiple strands.

8. Discussion

The radial pressure at the interface is converted to bond stress using a friction interface. The associated friction coefficient is a property of the type of prestressing steel and its surface condition. A coefficient of friction of 0.75 has been used to generate the theoretical curves. This value is used throughout the analysis. Usually all 3 cases defined in Fig. 4 (namely I, II, and III) appear as shown in Fig. 9 and 10. The fully cracked zone will not occur when the tendons have a large cover. The shape of the prestress build-up profile can have concave, linear or convex portions as shown in Fig. 9 and 10. These zones are shown in Fig. 11 as a general diagram for clarity.

9. Conclusions

1. When a tendon is released, the concrete near the end of the tendon may become fully cracked. The concrete surrounding the tendon further along will be partially cracked and near the end of the transfer zone will be uncracked.
2. The shapes of the prestress build-up profiles vary throughout the transfer zone. These can have concave, linear or convex portion as shown.
3. Computations show that the partially cracked region is the most important one as most of the prestress transfer occurs in this zone.
4. The expected width of cracks in the transfer zone falls within what might be called the "cohesive cracking" range. Tensile stresses therefore continue to be transferred across these cracks.
5. Modelling of the cohesive cracking zone as an anisotropic material has been achieved and a solution derived using a numerical procedure.
6. Based on the theoretical analysis that has been developed, the bond strength depends on the concrete strength, the tendon stress, size, cover and spacing, and the location along the transfer length.
7. The theory developed in this work successfully describes the stress and strain state in the prestress transfer bond region. The effect of changes to one or more of the variable can therefore be predicted.
8. The proposed procedure seems to unravel the influences of the parameters that affect stress development in the prestress transfer zone

References

1. Hoyer, E., und Friedrich, E., "Beitrag zur Frage der Stahlsaitenbeton", Berlin, Leipzig, Elsner, Haftspannung in Eisenbeton-bauteilen, Beton und Eisen, Vol 38/6 - 1939, pp. 107-110.
2. Weerasekera, I.R.A., "Transfer and Flexural Bond in Pretensioned Prestressed Concrete", Ph.D. Thesis, Department of Civil Engineering, The University of Calgary, Calgary, Canada 1991, pp 1-322.
3. Tepfers, R., "Cracking of Concrete Cover along Anchored Deformed Reinforcing Bars", Magazine of Concrete Research, Vol.31, No. 106, March 1979, pp 3-12
4. Hillerborg, A., "Analysis of One Single Crack", Fracture Mechanics of Concrete, Developments in Civil Engineering, 7, Editor F.H.Wittmann, Elsevier Science Publishers, Amsterdam, The Netherlands, 1983, pp 233- 249.
5. Weerasekera, I.R.A., and Loov, R.E., "Prestressed Transfer Length Predicted by Cohesive Crack Model", Proceedings of the International Conference on Micro- Mechanics of Failure of Quasi-Brittle Materials, Albuquerque, New Mexico, USA, June, 1990, pp 417-426.
6. Gopalratnam, V.S., and Shah, S.P., "Softening Response of Plain Concrete in Direct Tension", ACI Journal, Vol.82, No.3, May-June 1985, pp.310-323.
7. Kreyszig, E., "Advanced Engineering Mathematics", 5th Edition, John Wiley & Sons, New York, 1983, pp 1097.

NOTATION

A_b = area of prestressing steel
 c = radius of outer surface concrete
 c_y = concrete cover to surface of prestressing strand
 d_b = strand diameter
 E = modulus of elasticity
 E_c = modulus of elasticity of concrete
 E_p = modulus of elasticity of prestressing steel
 E_r = modulus of elasticity in the radial direction
 E_z = modulus of elasticity in the axial direction

E_θ = circumferential modulus of elasticity of cracked concrete
 f_{ce} = longitudinal stress in concrete
 f_{ci}' = strength of concrete at transfer
 f_{cz} = axial stress in the concrete at a distance z from the free end
 f_{pj} = prestress prior to transfer
 f_{pi} = initial prestress
 f_{pz} = axial stress in the prestressing steel at a distance z from the free end
 f_r = known stress in radial direction
 f_t = tensile strength of concrete
 f_{tr} = reduced tensile strength of concrete at the outer surface
 f_θ = known stress in circumferential direction
 l_t = transfer bond length
 L_c = extent of radial cracking
 p = interface pressure
 P_j = jacking prestressing force
 r = radial distance
 r_o = radius of the unstressed prestressing strand
 r_j = radius of the prestressed prestressing strand at jacking
 r_n = radial distance to section n
 R = radial distance to the inner boundary of elastic uncracked section
 t = step size
 u = radial displacement
 u_o = radial displacement of outer surface of steel cylinder
 u_j = radial displacement of inner surface of concrete cylinder
 u_n = radial displacement at section n
 u_j = radial displacement of inner surface of cracked concrete

z = distance from the free end of the prestressing steel
 ε = concrete strain
 ε_{cz} = axial strain of concrete at a distance z from free end
 ε_{pz} = axial strain of prestressing steel at a distance z from the free end
 ε_r = strain in the radial direction
 ε_z = strain in the axial direction
 ε_θ = strain in the circumferential direction
 μ = coefficient of friction
 ν = Poisson's ratio
 ν_c = Poisson's ratio for concrete
 ν_p = Poisson's ratio for prestressing steel
 σ_r = normal stress in radial direction
 σ_z = normal stress in axial direction
 σ_θ = hoop stress acting in the circumferential direction

TABLES AND FIGURES

List of Tables:

Table 1 - Properties of Test Beams

Table 2 - Part of Theoretical and Experimental Results

Table 3 - Variation of transfer length

List of Figures:

Fig. 1 - Relative Radial Displacement of Steel and Concrete

Fig. 2 - Isolated Steel Part from Sliced Element

Fig. 3 - Stresses Acting on Element

Fig. 4 - Possible Situations for Analysis

Fig. 5 - Definitions for Transfer Length

Fig. 6 - Theoretical and Experimental Curves for Beam A-1

Fig. 7 - Theoretical and Experimental Curves for Beam B-1

Fig. 8 - Theoretical and Experimental Curves for Beam C-1

Fig. 9 - Variation of Steel Stress in Transfer zone for 15.24 mm Strand with 20 MPa Concrete

Fig. 10 - Variation of Steel Stress in Transfer zone for 15.25 mm Strand with 40 MPa Concrete

Fig. 11 - Stress Build-up in the Transfer zone

Table 1 – Properties of Test Beams

Group	A			B		C	
Beam designation	A-1	A-2	A-3	B-1	B-2	C-1	C-2
Concrete Cylinder Strength * (MPa)	19.8	22.5	19.7	37.5	36.2	22.4	23.5
Strand Diameter (mm)	12.7			12.7		15.24	
Side Cover (mm)	40			40		39	
Bottom Cover (mm)	40			40		39	
Spacing (mm)	93			93		93	
Leg Width (mm)	185			185		185	
Flange width (mm)	400			400		400	
Flange Depth (mm)	300			300		300	
Total Depth (mm)	400			400		400	

* Age : At transfer 4 days

Table 2 – Part of Theoretical and Experimental Results

Group	No of Transfer Lengths Measured	Transfer Length (mm)			
		Theoretical	Experimental		
			Minimum	Average	Maximum
A	12	515	426	552	741
B	8	382	274	356	433
C	8	842	771	798	843

Note: Total number of results is 52.

Table 3 - Variation of transfer length

Case	d_b (mm)	c_y (mm)	f'_{ci} * (MPa)	f_{pj} (MPa)	f_{pi} (MPa)	f_{ce} (MPa)	l_t (mm)	l_t/d_b
1	15.24	20	20	1300	1214	6	1956	128
2	15.24	20	20	1300	1130	12	1982	130
3	15.24	20	20	900	817	6	918	60
4	15.24	20	20	900	737	12	935	61
5	15.24	40	20	1300	1214	6	863	57
6	15.24	40	20	1300	1130	12	876	57
7	15.24	40	20	900	817	6	469	31
8	15.24	40	20	900	736	12	477	31
9	15.24	20	40	1300	1239	6	1377	90
10	15.24	20	40	1300	1178	12	1388	91
11	15.24	20	40	900	841	6	647	42
12	15.24	20	40	900	783	12	654	43
13	15.24	40	40	1300	1239	6	609	40
14	15.24	40	40	1300	1178	12	614	40
15	15.24	40	40	900	841	6	332	22
16	15.24	40	40	900	783	12	336	22
17	9.53	20	20	1300	1214	6	479	50
18	9.53	20	20	1300	1129	12	483	51
19	9.53	20	20	900	814	6	306	32
20	9.53	20	20	900	731	12	312	33
21	9.53	40	20	1300	1213	6	294	31
22	9.53	40	20	1300	1130	12	302	32
23	9.53	40	20	900	814	6	267	28
24	9.53	40	20	900	731	12	271	28
25	9.53	20	40	1300	1239	6	335	35
26	9.53	20	40	1300	1178	12	338	35
27	9.53	20	40	900	839	6	216	23
28	9.53	20	40	900	779	12	218	23
29	9.53	40	40	1300	1238	6	210	22
30	9.53	40	40	1300	1178	12	212	22
31	9.53	40	40	900	839	6	192	20
32	9.53	40	40	900	779	12	194	20

* Cylinder strength

Figure 1: Relative Radial Displacement of Steel and Concrete

Figure 2: Isolated Steel Part from Sliced Element

Figure 3: Stresses Acting on Element

Figure 4: Possible Situations for Analysis

Figure 5: Definitions for Transfer Length

Figure 6 - Theoretical and Experimental Curves for Beam A-1

Figure 7 - Theoretical and Experimental Curves for Beam B-1

Figure 8 - Theoretical and Experimental Curves for Beam C-1

Figure 9 – Variation of Steel Stress in Transfer zone for 15.24 mm Strand with 20 MPa Concrete

Note – The numbers referred to in the curves are defined in Table 3.

* Cylinder Strength

Figure 10 – Variation of Steel Stress in Transfer zone for 15.25 mm Strand with 40 MPa Concrete

Note – The numbers referred to in the curves are defined in Table 3.

* Cylinder Strength

Figure 11 - Stress Build-up in the Transfer zone